# High Spin States in Nuclei: Exotic Quantal Rotation II 

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## "standard" $\gamma$-ray spectroscopic measurements:

- $\gamma-\gamma$ coincidences $\rightarrow$ level structure
- Angular Distributions/Correlations
$\rightarrow$ angular momentum transfer $\Delta L$
- Linear Polarizations $\rightarrow$ parity

Lifetimes $\rightarrow$ transition probabilities
$E \Delta L$ : Electric, $\pi=(-1)^{\Delta L} ;$ E2
$\mathrm{M} \Delta L$ : Magnetic, $\pi=(-1)^{\Delta L+1} ; \mathrm{M} 1$
$B(E 2) \rightarrow Q, \beta$


$8^{+}$

## E2 transitions

$\mathrm{E}_{I}=\left(\hbar^{2} / 2 \mathcal{J}\right) /(1+1) ; \mathrm{E}_{\gamma}=[](4 I-2)$
$\Delta \mathrm{E} \gamma=$ Constant $\rightarrow$ "picket fence"

## Magnetic and Anti-Magnetic Rotation



Ferromagnet

Magnetic rotor


Anti-Ferromagnet

G. Baldsiefen et al., Nucl. Phys. A 574, 521 (1994).


FIG. 18. Magnetic rotation. The high- $j$ proton particles and neutron holes form current loops embedded in the near spherical mass distribution of the nucleus. These current loops as well as the associated transverse magnetic moment $\mu_{\perp}$ allow us to specify the angle $\psi$ of rotation around the axis $\vec{J}$. The total angular momentum $J$ increases by the gradual alignment of the particle and hole angular momenta $\vec{j}_{p}$ and $\vec{j}_{n}$. This is called the shears mechanism. The interaction due to shape polarization tries to keep $\vec{j}_{p}$ and $\vec{j}_{n}$ at a right angle, like the spring of a pair of shears. From Clark and Wadsworth, 1998.

[^0]
## Tilted-axis Cranking


$\mathrm{Fig}_{\mathrm{g}} 1$. Orientation of the axis of rotation with respect to the principal axes of the triaxial density distribution. The long, intermediate and short axes ar labelled by $l$, $i$ and s , respectively


Fig. 2. Angular momentum composition of the shears bands with the high- $j$ configuration $\pi h_{9 / 2}, \pi i_{13 / 2}, \pi s_{1 / 2}^{-2}, \nu i_{13 / 2}^{-2}$. Band 2 in ${ }^{199} \mathrm{~Pb}$ (cf. [6]) has this configuration combined with a quasineutron on a low- $j$ - $f$ porbital

## S. Frauendorf, Z Phys. A 358, 163 (1997)

| $\begin{gathered} \Delta I=2 \\ \text { ordinary bands } \end{gathered}$ | Characteristic of rotational bands | $\Delta \mathrm{I}=1$ <br> shears bands |
| :---: | :---: | :---: |
| gradual alignment of many short vectors <br> E2 | regular $E_{\gamma} \propto I$ <br> enhanced transitions | gradual alignment of few long vectors <br> M1 |
| electric quadrupole mass distribution classic and quantal | possibily to define the orientation (with respect to the a. $m$. vector) <br> large <br> isotropy broken "inertia" $\mathcal{J}^{(\lambda)}=\Delta \mathrm{I} / \Delta \mathrm{E}_{\mathrm{y}}$ | magnetic dipole current distribution <br> quantal |
| electric | rotation | magnetic |

Fig. 4. The relation between electric and magnetic rotation

Moments of Inertia, $\mathfrak{J}^{(2)}$, are substantial $\sim 20 \mathrm{MeV}^{-1}$ $B(M 1)$ values are very large (several $\mu_{N}{ }^{2}$ ) and decrease with spin.
Very small B(E2)s for crossover transitions ( $\beta<0.1$ ) $J^{(2) / B(E 2) ~ v e r y ~ l a r g e ~}$
(>100 MeV-1 $(\mathrm{eb})^{-2}$, compared with $\sim 10$ for "normal")


Fig. 3. Angular momentum and $B M 1$ values as functions of the rotational frequency for band 2 in ${ }^{199}$ Pb. For the configuration of. Fig 2. Full lines are the TAC calculation. Points are the experimental data from $[6,8,9]$. In $(3)$ the experimental value of 0.5 is taken for the spin attenuation factor of the $i_{13 / 2}$ neutrons

## Anti-Magnetic Rotation




FIG. 2. Level scheme of ${ }^{100} \mathrm{Pd}$ obtained in this work. The widths of the arrows are proportional to the relative intensities of the transitions. For the transitions proposed as corresponding to antimagnetic rotation (AMR), intensities relative to the $665-\mathrm{keV}$ line are given in parentheses next to the transition energies. In general, the uncertainties in transition energies are $\sim 0.5 \mathrm{keV}$ and in the intensities $\sim 10 \%$. The moments of inertia, $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$, for the AMR transitions are shown in the inset.

## S. Zhu et al., Phys. Rev. C 64, 041302 (R) (2001)



ENERGY


## S. Zhu et al., Phys. Rev. C 64, 041302 (R) (2001)



## A.J. Simons et al., Phys. Rev. Lett. 91, 162501 (2003)


V. Singh et al., J. Phys. G 44, 075105 (2017)


S. Frauendorf, Y. Gu, and J. Sun, Int. Mod. Phys. E 20, 465 (2011).

## Tidal Waves in Nuclei:

The concept of nuclear tidal waves is based on the postulate that the yrast states (states of maximal angular momentum) in a phonon multiplet represent waves that travel over the nuclear surface akin to tidal waves over the ocean surface.

This nomenclature accounts for the observation that the yrast states in vibrational nuclei have a rotationallike structure for which a sound theoretical basis, especially at higher spins, has been lacking so far.


## Minimal angular momentum

$$
R(\vartheta, \varphi, t)=R_{o}\left[1+\beta \cos (\Omega t) Y_{20}(\vartheta, \varphi=0)\right]
$$

## Maximal angular momentum

$$
R(\vartheta, \varphi, t)=R_{o}\left(1+\sqrt{2} \beta \sin \gamma \cos (2 \varphi-\Omega t) Y_{22}(\vartheta, \varphi=0)\right)
$$

animations courtesy of S. Frauendorf

In rigid rotation, the energy and angular momentum increase with increase in frequency; the shape (moment of inertia) remains unchanged.
$B(E 2)$ ~ Constant

In tidal wave, the energy and angular momentum increase with the amplitude of the wave (change in shape); the frequency remains constant.

$$
B(E 2) \rightarrow J
$$



S. Frauendorf, Y. Gu, and J. Sun, Int. Mod. Phys. E 20, 465 (2011).


102Pd

A. D. Ayangeakaa et al.,

Phys. Rev. Lett. 110, 102501 (2013)

## MEASURED LIFETIMES AND ELECTROMAGNETIC TRANSITION PROBABILITIES FOR THE YRAST BAND IN ${ }^{102} \mathrm{Pd}$.

| $\mathrm{E}_{\gamma}(\mathrm{MeV})$ | $\mathrm{I}_{i}^{\pi} \rightarrow \mathrm{I}_{f}^{\pi}$ | $\tau(\mathrm{ps})$ | $B(E 2)\left(\mathrm{e}^{2} \mathrm{~b}^{2}\right)$ | $Q_{t}(\mathrm{eb})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.836 | $6^{+} \rightarrow 4^{+}$ | $1.15(32)$ | $0.174(48)$ | $2.36(33)$ |
| 0.902 | $8^{+} \rightarrow 6^{+}$ | $0.720(42)$ | $0.190(11)$ | $2.41(07)$ |
| 0.980 | $10^{+} \rightarrow 8^{+}$ | $0.409(28)$ | $0.221(15)$ | $2.56(09)$ |
| 1.062 | $12^{+} \rightarrow 10^{+}$ | $0.252(18)$ | $0.240(17)$ | $2.65(09)$ |
| 1.083 | $14^{+} \rightarrow 12^{+}$ | $0.201(11)$ | $0.272\left({ }_{-22}^{+15}\right)$ | $2.80\left({ }_{-13}^{+08}\right)$ |
| 1.104 | $16^{+} \rightarrow 14^{+}$ | $0.180(21)^{2}$ | $0.277\left({ }_{-42}^{+33}\right)$ | $2.81\left({ }_{-21}^{+17}\right)$ |


A. D. Ayangeakaa et al.,

Phys. Rev. Lett. 110, 102501 (2013)




In the simplest approximation of pure harmonic motion, the energy and the angular momenta of the yrast states is given by:

$$
\begin{equation*}
E_{n}=n \omega_{0} \quad I=2 n \tag{1}
\end{equation*}
$$

where $\boldsymbol{n}$ is the number of phonons with energy $\omega_{0}$.
It is of interest to consider the effects of anharmonic terms [3,4] arising from the interaction between phonons. To lowest order, Eq. (1) gets modified as follows:

$$
\begin{equation*}
E_{n} \approx n \omega_{0}+\frac{1}{2} V_{0} n(n-1)+\ldots \tag{2}
\end{equation*}
$$

Eq. (2) can be rewritten in terms of angular momentum:

$$
E(I) \approx\left(\frac{\omega_{0}}{2}-\frac{V_{0}}{4}\right) I+\frac{1}{8} V_{0} I^{2}+\ldots
$$

$$
E(I) \approx a I+b I^{2}+\ldots
$$

from which the rotational frequency and the moment of inertia can be derived

$$
\begin{align*}
& \omega=\frac{\partial E(I)}{\partial I} \approx a+2 b I+\ldots \\
& \mathfrak{F}=\frac{I}{\omega} \approx \frac{I}{a+2 b I} \tag{3}
\end{align*}
$$

Along the same lines, the transition probabilities can be parametrized in the form [4]:

$$
\begin{equation*}
B(E 2) \approx c I+d I^{2}+e I^{3}+\ldots \tag{4}
\end{equation*}
$$

A.O. Macchiavelli et al., Phys. Rev. C 90, 047304 (2014).


FIG. 1. Top: Moment of inertia in the yrast band in ${ }^{102} \mathrm{Pd}$ and the results of Eq. (5). Bottom: $B(E 2)$ values and Eq. (6). The harmonic and rotational limits are shown in dashed and dotted lines
A. O. Macchiavelli et al.,

Phys. Rev. C 90, 047304 (2014).
$\checkmark$ Measured B(E2)'s of the yrast band in ${ }^{102} \mathrm{Pd}$ vary linearly with spin
conforms to expectation from a tidal wave picture rotating condensate of 7 aligned d-bosons
$\checkmark$ Alternative explanation in terms of GCM cannot simultaneously describe both energies and B(E2)'s.
$\checkmark$ A simple phenomenological phonon model, with anharmonic terms reproduces the observed data, providing an independent confirmation of the multi-phonon picture associated with tidal waves..
> A.D. Ayangeakaa et al., Phys. Rev. Lett. 110, 102501 (2013).
> A.O. Macchiavelli et al., Phys. Rev. C 90, 047304 (2014).

## What is Chirality?

The word chiral comes from the Greek "chaire" (hand). So, "Chirality" stands for "handedness"


## Chirality is quite common in nature:

All terrestrial life uses only right-handed sugars and left-handed amino acids.

More than 50 percent of the world's top 100 drugs are chiral, including familiar brand names such as Lipitor, Paxil, Zoloft and Nexium.

The classic, tragic example is the sedative thalidomide, made infamous during the 1960s. Though thalidomide's right-handed enantiomer helped fight morning sickness in pregnant women, its left-handed version caused severe birth defects.

In the case of the common pain-reliever ibuprofen, the molecule's right-handed version is simply 100 times less powerful than its left-handed twin.

## Chirality in Nuclei

$\gamma$-deformation, $\gamma \sim 30^{\circ}$

$$
\mathcal{J}_{s} \sim J_{l}<\mathcal{J}_{i}
$$

Configurations Particles: short-axis Holes: long-axis
Core: intermediateaxis


The left-handed configuration is converted into the right handed one by the time reversal operation


Fig. 1. (Color online.) A schematic picture for the aplanar rotation of a triaxial nucleus. The arrow $\boldsymbol{J}$ denotes the total angular momentum vector, and the short, intermediate, and long axes are denoted by $x, y$, and $z$, respectively.

P. W. Zhao, Phys. Lett. B 773, 1 (2017)



# A Triaxial Nucleus becomes Chiral 

if it rotates about an axis that lies outside the three planes spanned by the principal axes of it's triaxial ellipsoidal shape.


Chirality in Nuclei

> Two exactly identical bands with same Energy and Parity De-excitation of the partner bands in a very similar way via Electromagnetic radiation

Band A

S. Zhu et al., Phys. Rev. Lett. 91, 132501 (2003)

## 3D TAC Results




| $\hbar \omega[\mathrm{MeV}]$ | $\vartheta$ | $\varphi$ | $I(\omega)[\hbar]$ |
| :---: | :---: | :---: | :---: |
| 0.45 | $65^{\circ}$ | $0^{\circ}$ | 20.1 |
| 0.50 | $75^{\circ}$ | $28^{\circ}$ | 22.0 |
| 0.55 | $75^{\circ}$ | $37^{\circ}$ | 25.0 |
| 0.60 | $85^{\circ}$ | $47^{\circ}$ | 30.4 |

$\pi\left(h_{11 / 2}\right)^{2} v\left(h_{11 / 2}\right)^{-1}$

## ${ }^{135} \mathrm{Nd}$



FIG. 3. Chiral fingerprints: (a) excitation energy vs spin; (b) $S(I)$ vs spin; (c) $B(M 1) / B(E 2)$ and $B(M 1)_{\text {in }} / B(M 1)_{\text {out }}$.


FIG. 2. Partial level scheme showing the $\pi g_{9 / 2} \otimes \nu h_{11 / 2}$ chiral bands in ${ }^{104} \mathrm{Rh}$.


Fig. 1. (Color online.) Partial level scheme for ${ }^{80} \mathrm{Br}$ obtained in the present work. New observed transitions are indicated by stars and red lines.


FIG. 1. Partial level scheme showing the pair of bands in ${ }^{198} \mathrm{Tl}$.


## $\pi\left[\left(1 g_{7 / 2}\right)^{-1}\left(1 h_{11 / 2}\right)^{1}\right] \otimes \nu\left(1 h_{11 / 2}\right)^{-1}$ $\pi\left(1 h_{11 / 2}\right)^{2} \otimes \nu\left(1 h_{11 / 2}\right)^{-1}$

## A. D. Ayangeakaa et al., Phys. Rev. Lett. 110, 172504 (2013)


A. D. Ayangeakaa et al.,

Phys. Rev. Lett. 110, 172504 (2013)

I. Kuti et al., Phys. Rev. Lett. 113, 032501 (2014)


In contrast with the multiple chiral doublets predicted in Ref. [22] and experimentally reported in ${ }^{133} \mathrm{Ce}$ [26], the observed $\mathrm{M}_{\chi} \mathrm{D}$ in the negative-parity bands of ${ }^{103} \mathrm{Rh}$ is built from the first and second doublets of the same configuration. Although the two doublets belong to the same configuration, the angular momenta couplings for the two pairs of chiral partners are different. This fact is reflected by the different
I. Kuti et al., Phys. Rev. Lett. 113, 032501 (2014)

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# Multiple chirality in nuclear rotation: A microscopic view 

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#### Abstract

Covariant density functional theory and three-dimensional tilted axis cranking are used to investigate multiple chirality in nuclear rotation for the first time in a fully self-consistent and microscopic way. Two distinct sets of chiral solutions with negative and positive parities, respectively, are found in the nucleus ${ }^{106} \mathrm{Rh}$. The negative-parity solutions reproduce well the corresponding experimental spectrum as well as the $B(M 1) / B(E 2)$ ratios of the transition strengths. This indicates that a predicted positive-parity chiral band should also exist. Therefore, it provides a further strong hint that multiple chirality is realized in nuclei. © 2017 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP ${ }^{3}$.


ありがとう
धन्य वाद
Thanks！


The Question Kitten


[^0]:    S. Frauendorf, Rev. Mod. Phys. 73, 463 (2001)

