

Theoretical Study of Superheavy Nuclei

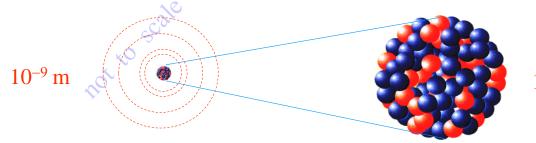
--- Structure Properties & Synthesis Mechanism ---

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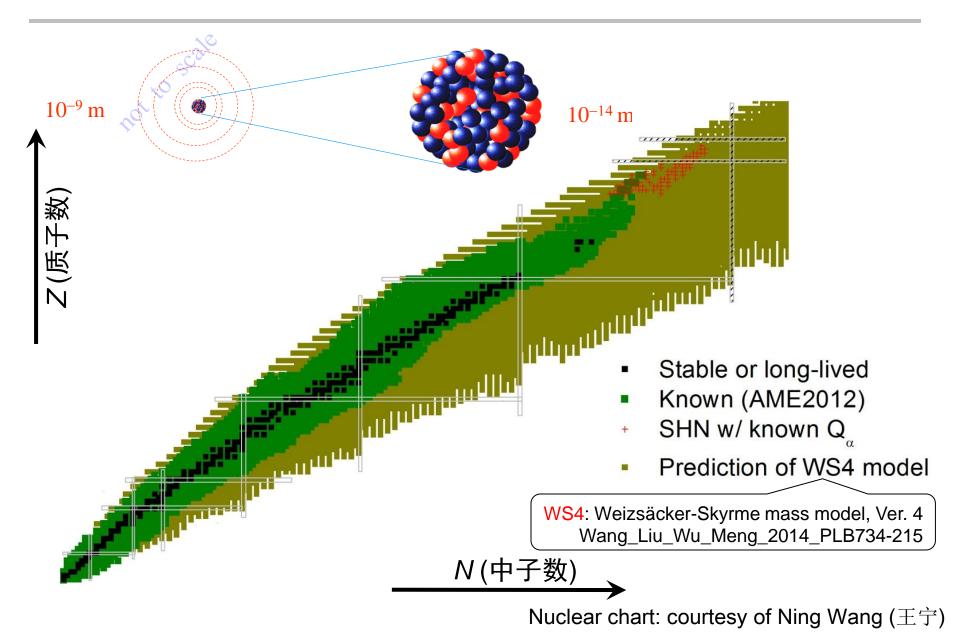
> Supported by: NSFC, CAS & MOST HPC Cluster of KFTP/ITP-CAS ScGrid of CNIC-CAS

Atomic nucleus & nuclide

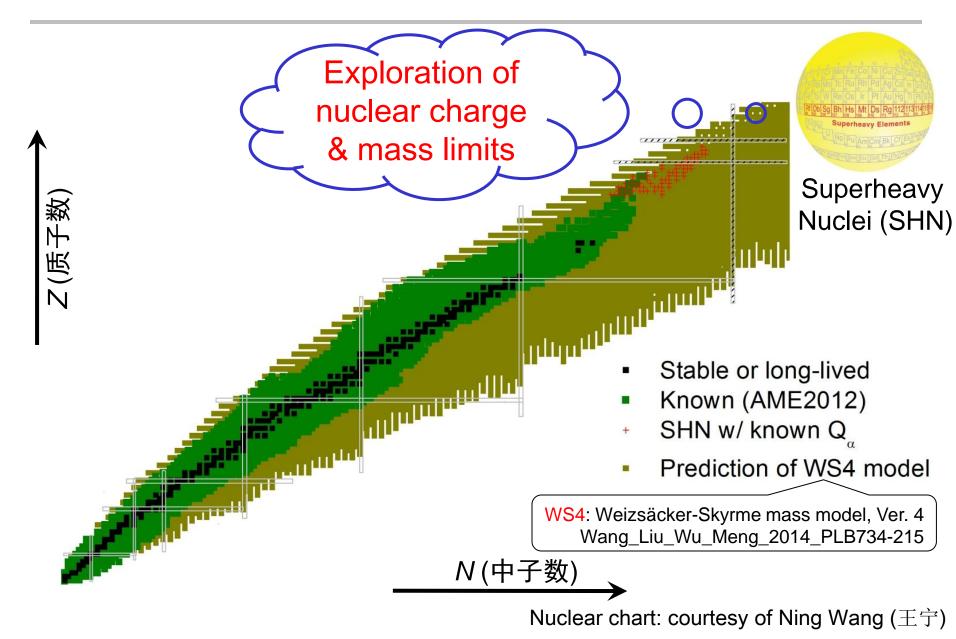


 $10^{-14}\,{\rm m}$

Chart of nuclides



Extension of chart of nuclides: Superheavy nuclei



Main content of lectures

- Predictions, experimental progress & challenges
- □ Structure, decay & fission properties
- Synthesis mechanism: Heavy-ion fusion & multinucleon transfer reactions

Further readings:

- Special Issue on Superheavy Elements, Nucl. Phys. A 944 (2015)
- Lu, Zhao & SGZ, Chapter 5 in Relativistic Density Functional for Nuclear Structure (World Scientific, 2016, Editor: Jie Meng)
- Ackermann & Theisen, Phys. Scr. 92 (2017) 083002
- Giuliani et al., Rev. Mod. Phys. 91 (2019) 011001

Lecture 1

□ Predictions of the island of stability of SHN

□ Experimental progress on synthesis of SHN

□ Challenges in synthesizing SHN

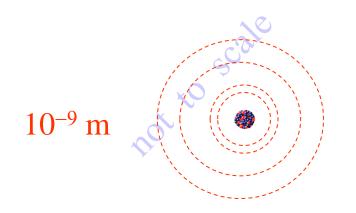
Lecture 1

Predictions of the island of stability of SHN

Experimental progress on synthesis of SHN

□ Challenges in synthesizing SHN

Nuclear charge limit on atomic level

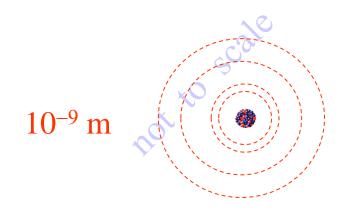


Bohr's model: Velocity of innermost electron

- ➤ Hydrogen: $v_1 = \alpha c$
- > Heavy atoms: $v_1 \sim Z\alpha c$, Z < 137

Greiner & Reinhardt 1994, QED Indelicato 2013, Nature 498, 40-41

Nuclear charge limit on atomic level



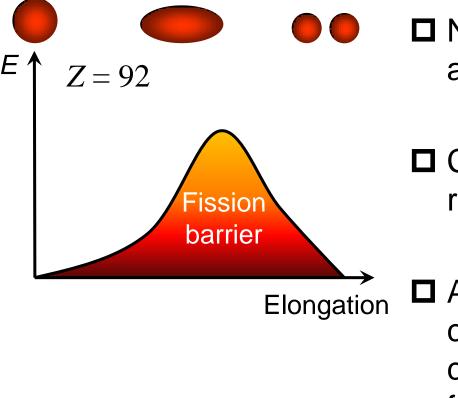
Bohr's model: Velocity of innermost electron

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Greiner & Reinhardt 1994, QED Indelicato 2013, Nature 498, 40-41

- QED: Energy of innermost electron
 - Point-like nucleus:
 - $E_1 \sim m_e c^2 [(1 (Z\alpha)^2)^{1/2} 1], Z < 137$
 - > Finite-size nucleus: Z < 173

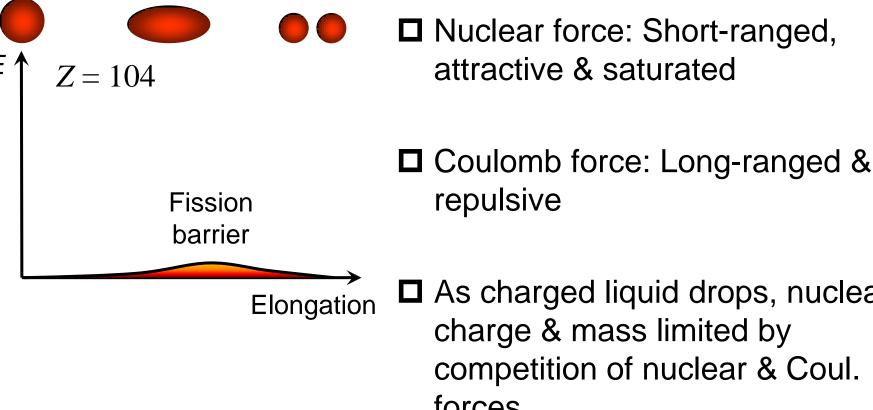
Nuclear charge & mass limits on nuclear level



- Nuclear force: Short-ranged, attractive & saturated
- Coulomb force: Long-ranged & repulsive

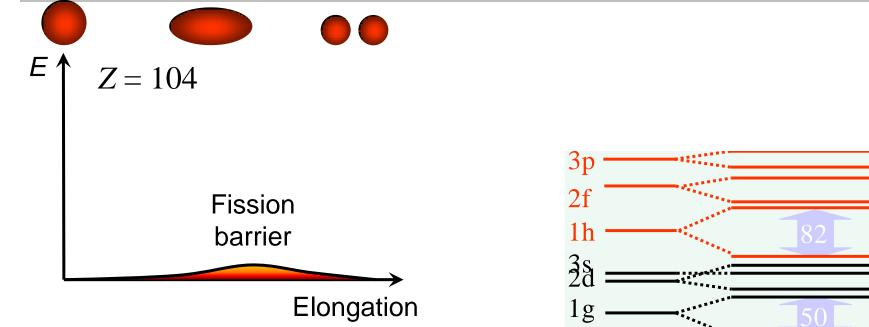
As charged liquid drops, nuclear charge & mass limited by competition of nuclear & Coul. forces

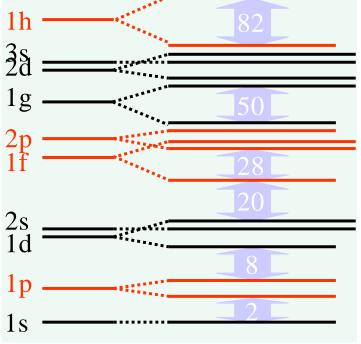
Nuclear charge & mass limits on nuclear level



□ As charged liquid drops, nuclear charge & mass limited by competition of nuclear & Coul. forces

Quantum shell effects





Single nucleon potential, spectra & magicities

- □ Harmonic oscillator & square well
- Woods-Saxon
- Self-consistent (self-bound)



Eugene Paul Wigner



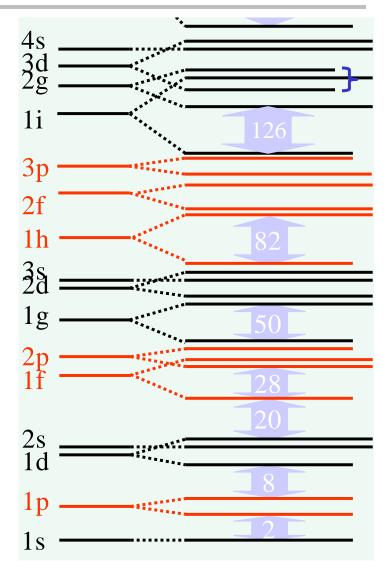


J. Hans D. Jensen

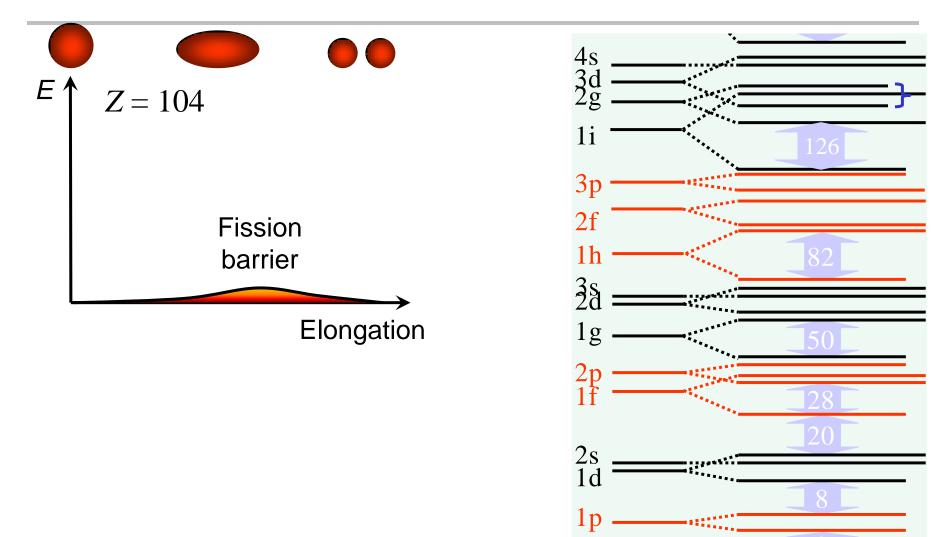
Nobel Prize of Physics (1963)

Maria Goeppert

Mayer

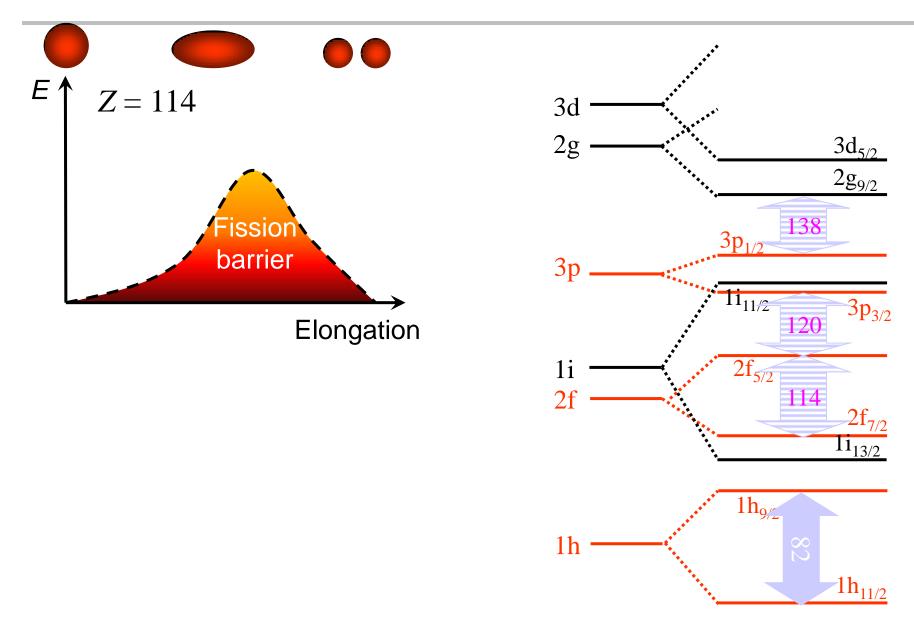


Quantum shell effects

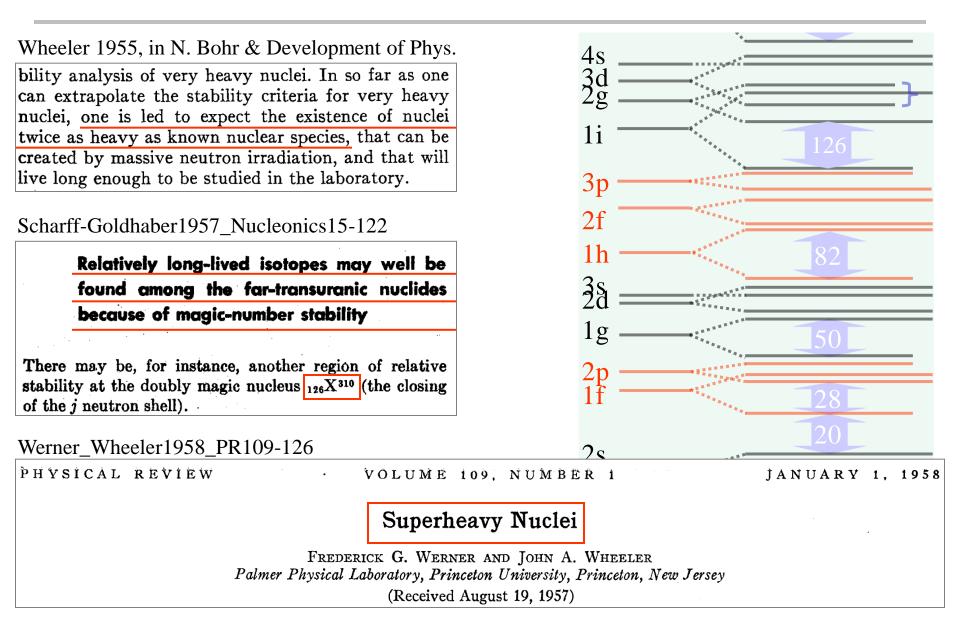


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Quantum shell effects \Rightarrow **Superheavy nuclei**



Before 1966: Extrapolation of shell structrure



1966: Semi-quantitative shell effects

1.E.2: 1.D.4 Nuclear Physics 81 (1966) 1–60; C North-Holland Publishing Co., Amsterdam

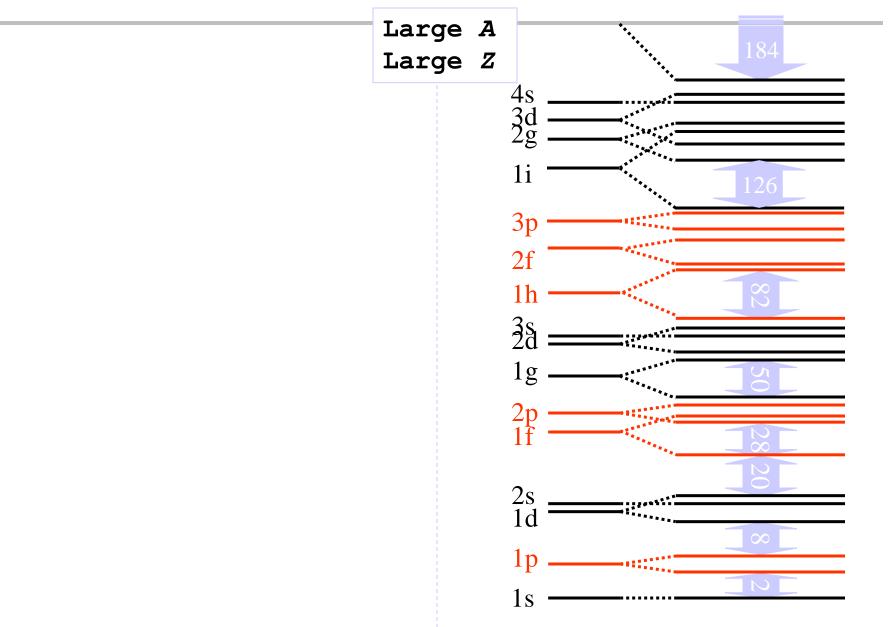
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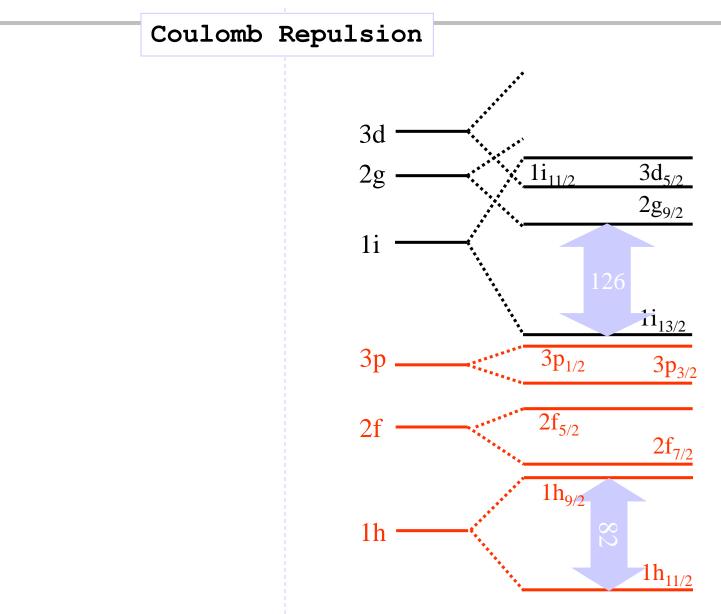
NUCLEAR MASSES AND DEFORMATIONS

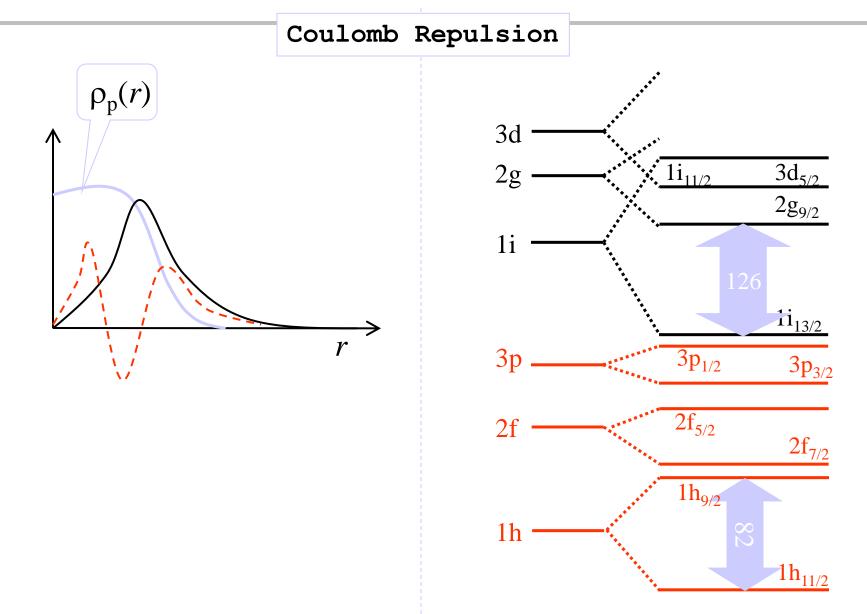
WILLIAM D. MYERS and WLADYSLAW J. SWIATECKI Lawrence Radiation Laboratory, University of California, Berkeley, California[†]

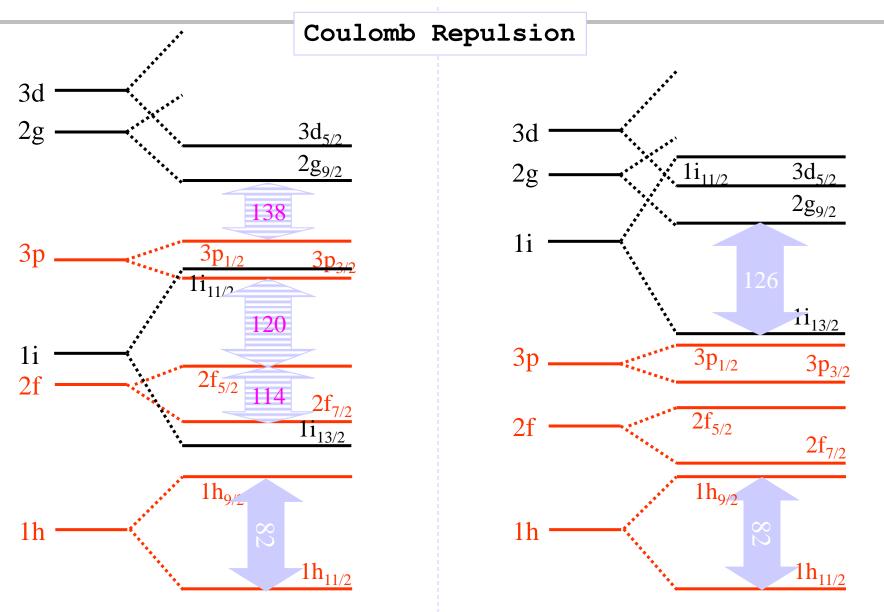
Received 7 September 1965

half-life. This is illustrated in fig. 2 where we have plotted the deformation energy predicted by our mass formula for the case Z = 126, N = 184. This nucleus has a fissility parameter x = 1.05; as a result, in the absence of shell effects, it would have a vanishing barrier against fission and a spontaneous fission half-life of the order of nuclear collective oscillations or 10^{-22} sec. Because of the assumed doubly magic number, however, the ground-state mass of this nucleus would be depressed (by about 10.2 MeV according to our formula). Since this depression is, according to our treatment, a rapidly decreasing function of deformation, there results a considerable barrier against fission, with a height of 9.0 MeV: the extra binding associated with the doubly magic number has stabilized the otherwise highly fissile nucleus. An









1966 & after: Quantitative shell effects

- □ Closed shells for Z > 82 and N > 126 in a diffuse potential well > A Woods-Saxon potential Sobiczewski...1966_PhysLett22-500 > Z = 114 & N = 184; $B_f = 10 \text{ MeV}$
- Predictions of new magic regions and masses for super-heavy nuclei from calculations with realistic shell model single particle Hamiltonian
 - ➤A non-local potential

Meldner1967_ArkivFysik36-593

>*Z* = 114 & *N* = 184

1967: A different opinion

VOLUME 18, NUMBER 17

PHYSICAL REVIEW LETTERS

24 April 1967

SHAPE OF HEAVY NUCLEI*

Philip J. Siemens[†] and H. A. Bethe Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received 10 March 1967)

The energy reduction for this semimagic nucleus is likely to be considerably less than for Pb²⁰⁸ for which it is about 15 MeV. On the other hand, without shell structure, the elongated shape has an energy about 18 MeV less than the sphere for Z = 114. We therefore believe that shell effects are unlikely to make the nucleus Z = 114 stable.

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□ More quantitative investigations needed !!!

Single particle shell structure

How much stability the closed shell brings

1966 & after: Quantitative shell effects

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A non-local potential

Meldner1967_ArkivFysik36-593

>*Z* = 114 & *N* = 184

Nilsson potential; potential energy surface with shell corrections calculated by the Strutinsky method Nilsson...1968 NPA115-545

 $> B_{\rm f} = 10 \text{ MeV } \& T_{1/2}(\text{s.f.}) \sim 10^{19} \text{ years}$

- □ "On the stability of superheavy nuclei against fission"
 - Nilsson potential; potential energy surface with rotational invariance
 - $> T_{1/2}(s.f.) \sim 10^{0-17} \text{ years}$ Mosel_Greiner1969_ZPA226-261

More investigations

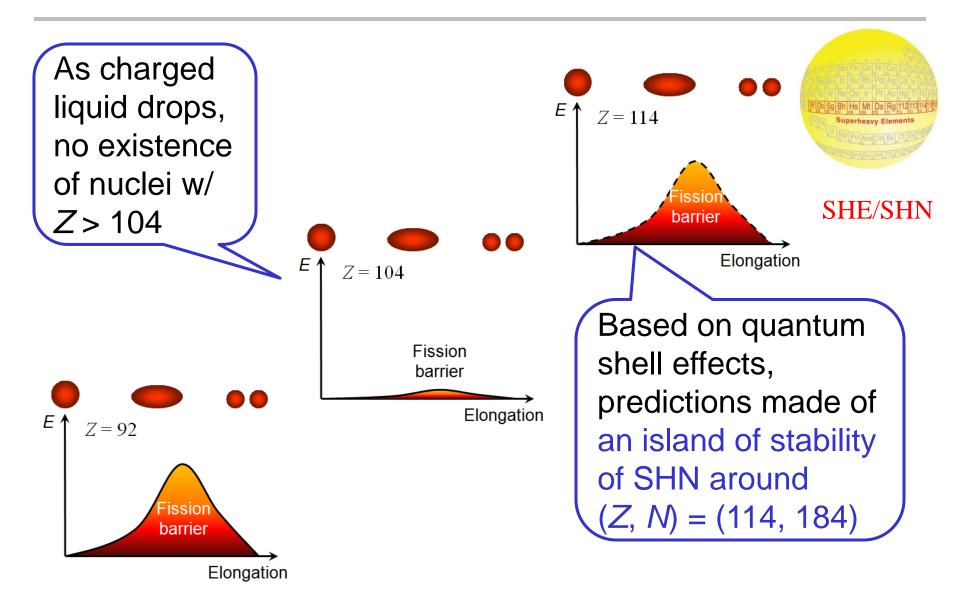
Not the end yet ! For the location of the island of stability, more different predictions:

- □ Macroscopic-microscopic models
- Self-consistent approaches
 - ≻Non-relativistic
 - ➢Relativistic
- Ο...

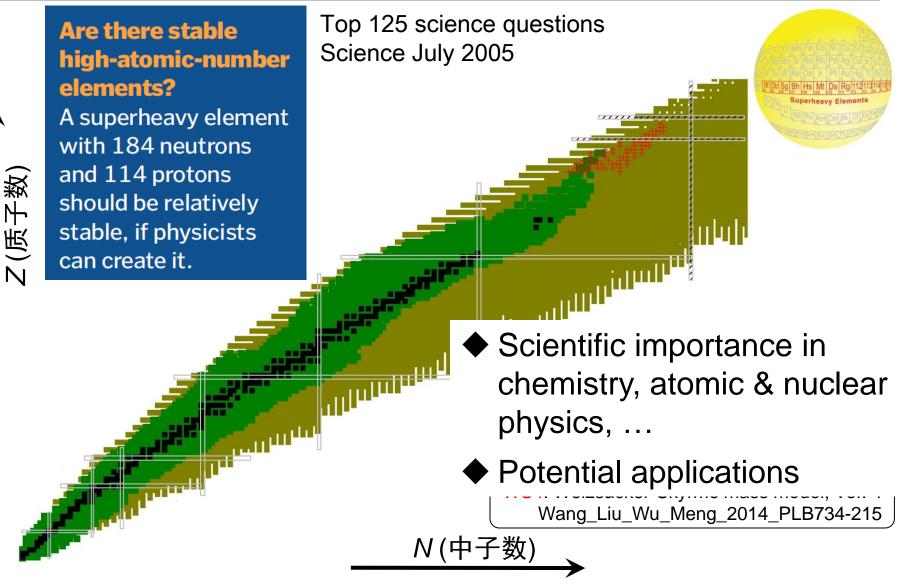
What are "Superheavy Nuclei" ?

- Can not exist if described as a charged liquid drop
- □ Stabilized by quantum shell effects

What are superheavy nuclei?

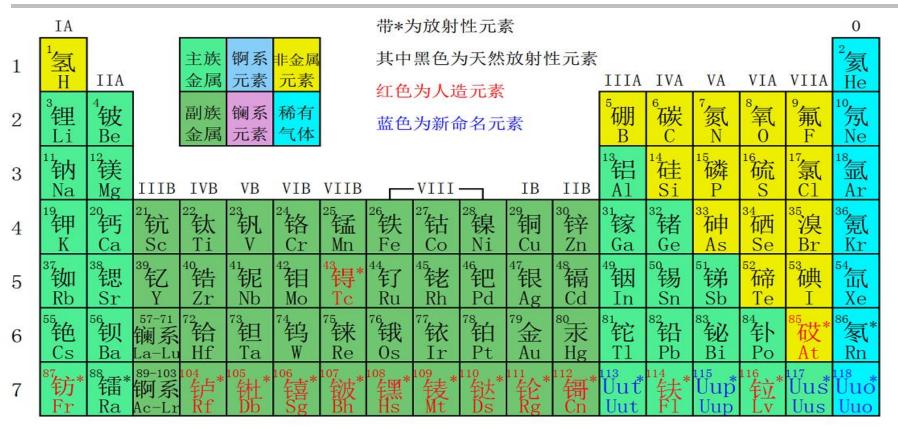


SHN: Charge & mass limits of nuclear existence



Nuclear chart: courtesy of Ning Wang (王宁)

Periodic table of elements: End?



SGZ, SHN & SHE, Physics 43 (2014) 817-825 (in Chinese)

| 镧系 | ⁵⁷ 、 領 La | ⁵⁸ 铈 Ce | ⁵⁹ 镨 Pr | ⁶⁰ 钕 Nd | ⁶¹ 征* Pm | ⁶² Sm | ⁶³ 有 Eu | ⁶⁴ 毛 Gd | ⁶⁵ 試 Tb | ⁶⁶ 宙 Dy | ⁶⁷ 秋 Ho | ⁶⁸ 年 Er | ⁶⁹ 铥 Tm | ⁷⁰ 镜 Yb | ⁷¹ 镥 Lu |
|----|----------------------------|----------------------------|--------------------------|--------------------------|------------------------|------------------------|--------------------------|--------------------------|------------------------------------|--------------------------|--------------------------|---------------------------|----------------------------|---------------------------|---------------------------|
| 锕系 | ⁸⁹ 何 Ac | ⁰⁰ 钍 [*] Th | ⁹¹ 镤 Pa | ⁹² 铀* U | ⁹³ 镎* Np | ⁹⁴ 标* Pu | ⁹⁵ 镅* Am | ⁹⁶ 锔* Cm | ⁹⁷ 锫 [*] Bk | ⁹⁸ 锎* Cf | ⁹⁹ 锿 Es | ¹⁰⁰ 馈 Fm | ¹⁰¹ 们* Md | ¹⁰² 锘 No | ¹⁰³ 铹 Lr |

Courtesy of Xu Meng (孟旭)

Lecture 1

□ Predictions of the island of stability of SHN

- Experimental progress on synthesis of SHN
- □ Challenges in synthesizing SHN

Expt. exploration of SHN & island of stability

- □ If half-life of SHN long enough (e.g, T_{1/2}~10⁸) & produced in nucleosynthesis
 Herrmann1979 Nature280-543
 - ➤SHE/SHN may exist in Nature
 - >No confirmed evidence, but efforts still being made
- ✓ 1860 (1861), Bunsen & Kirchhoff found cesium (rubidium) in mineral water (lepidolite) by using the flame spectroscopy
- ✓ 1868, Janssen & Lockyer recorded helium spectral line during solar eclipse
 ✓ …
- ✓ 1898, Curies discovered polonium (radium) in uranium ore pitchblende (uraninite) by identifying strong radioactivity

Expt. exploration of SHN & island of stability

- □ If half-life of SHN long enough (e.g, *T*_{1/2}~10⁸) & produced in nucleosynthesis
 - ➤SHE/SHN may exist in Nature
 - >No confirmed evidence, but efforts still being made
- In Lab, via heavy ion fusion reactions
 - ➢GSI in Darmstadt, Germany
 - Flerov Laboratory of Nuclear Reactions in Dubna, Russia
 - Lawrence Berkeley National Laboratory, USA
 - Lawrence Livermore National Laboratory, USA
 - ➢ RIKEN in Wako, Japan
 - ➢GANIL in Caen, France



Hofmann_Münzenberg2000_RMP72-733 Morita...2004_JPSJ73-2593 Oganessian...2007_JPG34-R165 Oganessian...2010_PRL104-142502 Zhang...2012_CPL29-012502

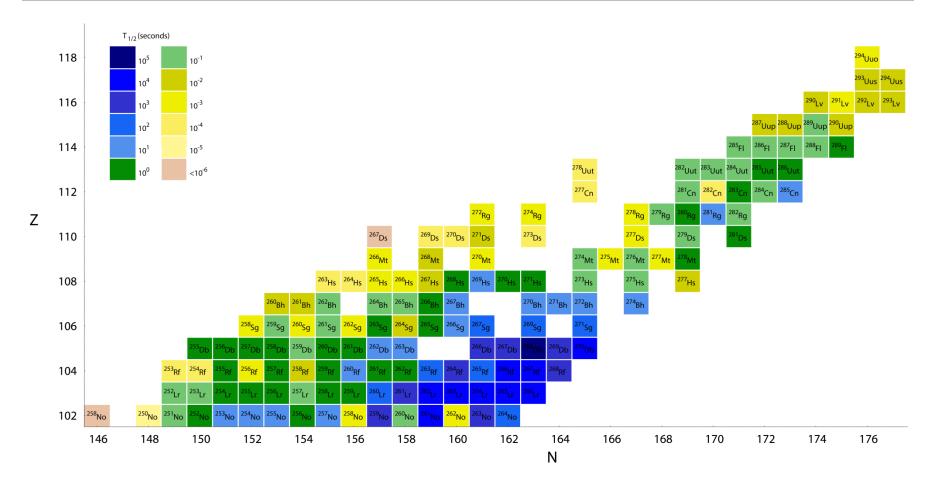
Synthesis of SHN in Labs

- □ Neutron capture followed by β decay
 > Up to fermium
- □ Light ions (H, D, T & α) as projectiles
 > Up to mendelevium
- Heavy ions as projectiles
 Cold fusion: ²⁰⁸Pb or ²⁰⁹Bi as targets, up to nihonium
 Hot fusion: ⁴⁸Ca as projectiles, up to oganesson





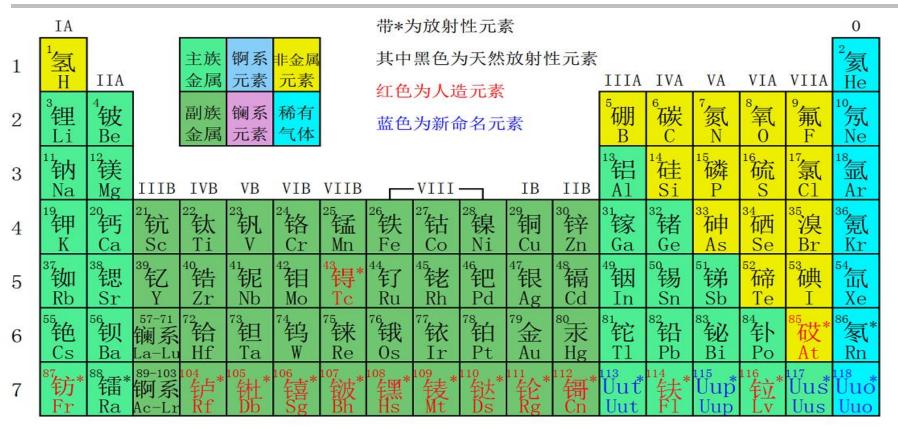
Expt. progress on synthesis of SHN



SHE with $Z \le 118$ have been synthesized & named Recent naming for elements with Z = 113, 115, 117 & 118

Courtesy of Kai Wen (温凯)

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|----|----------------------------|----------------------------|--------------------------|--------------------------|------------------------|------------------------|--------------------------|--------------------------|------------------------------------|------------------------------------|--------------------------|---------------------------|----------------------------|---------------------------|---------------------------|
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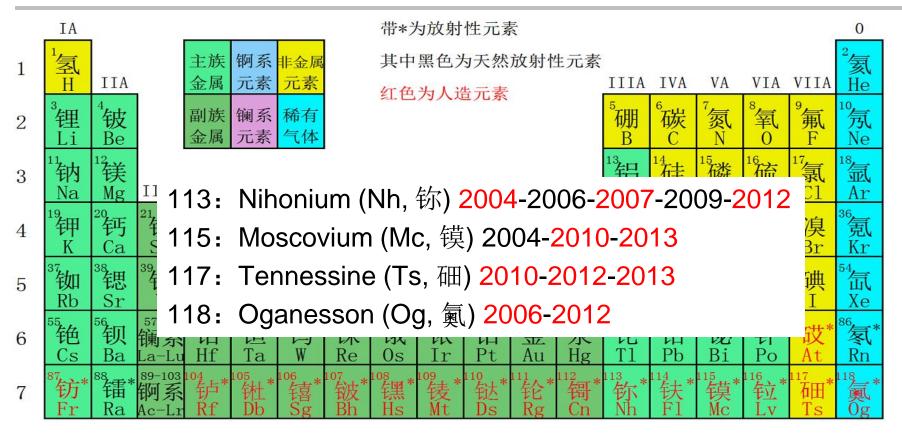
Courtesy of Xu Meng (孟旭)

Element 113

JWP ASSESSMENT: Three chains of ²⁷⁸113 observed by the RIKEN collaborations, the first in 2004 [9], the second in 2007 [10], and the third in 2012 [16], are now construed as being consistent. Firm connection to established nuclides is provided. The remaining criterion achieved for acknowledgement of discovery is an identification of *Z* which is now embodied in the cross reaction production and characterization of the chain beginning with ²⁶⁶Bh as found by the RIKEN collaboration in 2009 [14] and by Qin *et al.* in 2006 [12]. The Criteria for discovery have been met. Karol+2016_PureApplChem88-139

[9] K. Morita, K. Morimoto, D. Kaji, T. Akiyama, S. Goto, H. Haba, E. Ideguchi, R. Kanungo, K. Katori, H. Koura, H. Kudo, T. Morita+2004 Ohnishi, A. Ozawa, T. Suda, K. Sueki, H. Xu, T. Yamaguchi, A. Yoneda, A. Yoshida, Y.-L. Zhao. J. Phys. Soc. Jpn. 73, 2593 (2004). [10] K. Morita, K. Morimoto, D. Kaji, T. Akiyama, S. Goto, H. Haba, E. Ideguchi, K. Katori, H. Koura, H. Kikunaga, H. Kudo, T. Morita+2007 Ohnishi, A. Ozawa, N. Sato, T. Suda, K. Sueki, F. Tokanai, T. Yamaguchi, A. Yoneda, A. Yoshida. J. Phys. Soc. Jpn. 76, 045001 (2007). [11] P. A. Wilk, K. E. Gregorich, A. Türler, C. A. Laue, R. Eichler, V. Ninov, J. L. Adams, U. W. Kirbach, M. R. Lane, D. M. Lee, J. B. Patin, D. A. Shaughnessy, D. A. Strellis, H. Nitsche, D. C. Hoffman. Phys. Rev. Lett. 85, 2697 (2000). [12] Z. Qin, X. Wu, H. Ding, W. Wu, W. Huang, X. Lei, Y. Xu, X. Yuan, B. Guo, W. Yang, Z. Gan, H. Fan, J. Guo, H. Xu, G. Xiao. Nucl. Phys. Rev. 23, 400 (2006). Qin+2006 [13] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, R. N. Sagaidak, I. V. Shirokovsky, Yu. S. Tsyganov, A. A. Voinov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, V. G. Subbotin, A. M. Sukhov, K. Subotic, V. I. Zagrebaev, G. K. Vostokin, M. G. Itkis, R. A. Henderson, J. M. Kenneally, J. H. Landrum, K. J. Moody, D. A. Shaughnessy, M. A. Stover, N. J. Stover, P. A. Wilk. Phys. Rev. C 76, 011601(R) (2007). [14] K. Morita, K. Morimoto, D. Kaji, H. Haba, K. Ozeki, Y. Kudou, N. Sato, T. Sumita, A. Yoneda, T. Ichikawa, Y. Fujimori, S. Goto, E. Ideguchi, Y. Kasamatsu, K. Katori, Y. Komori, H. Koura, H. Kudo, K. Ooe, A. Ozawa, F. Tokanai, K. Tsukada, T. Yamagichi, Morita+2009 A. Yoshida. J. Phys. Soc. Jpn. 78, 064201 (2009). [15] Y. A. Akovali. Nucl. Data Sheets 94, 131 (2001). [16] K. Morita, K. Morimoto, D. Kaji, H. Haba, K. Ozeki, Y. Kudou, T. Sumita, Y. Wakabayashi, A. Yoneda, K. Tanaka, S. Yamaki, R. Sakai, T. Akiyama, S. Goro, H. Hasebe, M. Huang, T. Huang, E. Ideguchi, Y. Kasamatsu, K. Katori, Y. Kariya, H. Kikunaga, Morita+2012 H. Koura, H. Kudo, A. Mashiko, K. Mayama, S. Mitsuoka, T. Moriya, M. Murakami H. Murayama, S. Namai, A. Ozawa, N. Sato, K. Sueki, M. Takeyajma, F. Tokanai, T. Yamaguchi, A. Yoshida. J. Phys. Soc. Jpn 81, 103201 (2012).

Periodic table of elements: End?

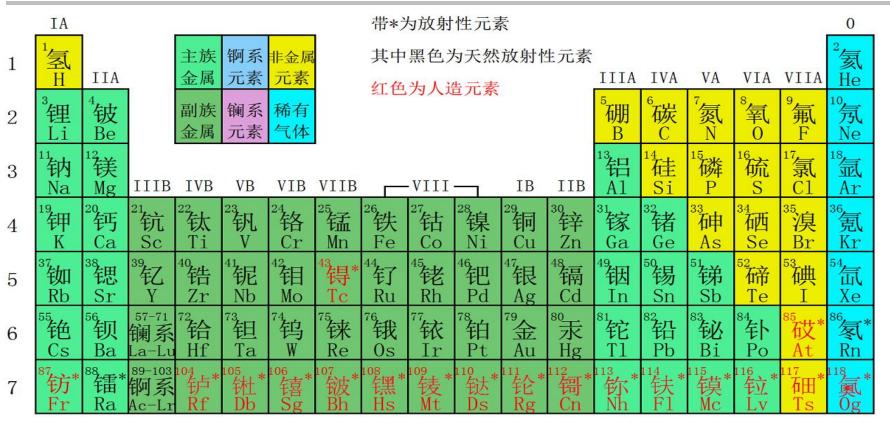


SGZ, SHN & New Elements, Nucl. Phys. Rev. 34 (2017) 318-331 (in, Chinese)

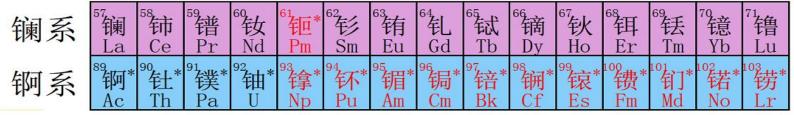


Courtesy of Xu Meng (孟旭)

Periodic table of elements: End?



SGZ, SHN & New Elements, Nucl. Phys. Rev. 34 (2017) 318-331 (in, Chinese)



Courtesy of Xu Meng (孟旭)

Chinese names of new elements

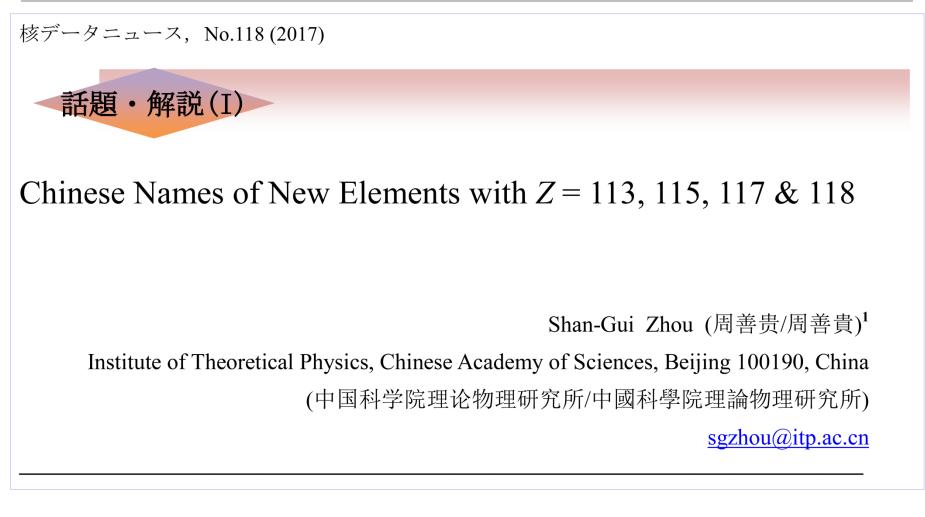
□ 115: Moscovium (Mc)

➤ Proposal of Chinese name: 镆

- □ 117: Tennessine (Ts)
 > Proposals of Chinese name: 础 & 磌
- □ 118: Oganesson (Og)
 > Proposals of Chinese name: 氣 & 気

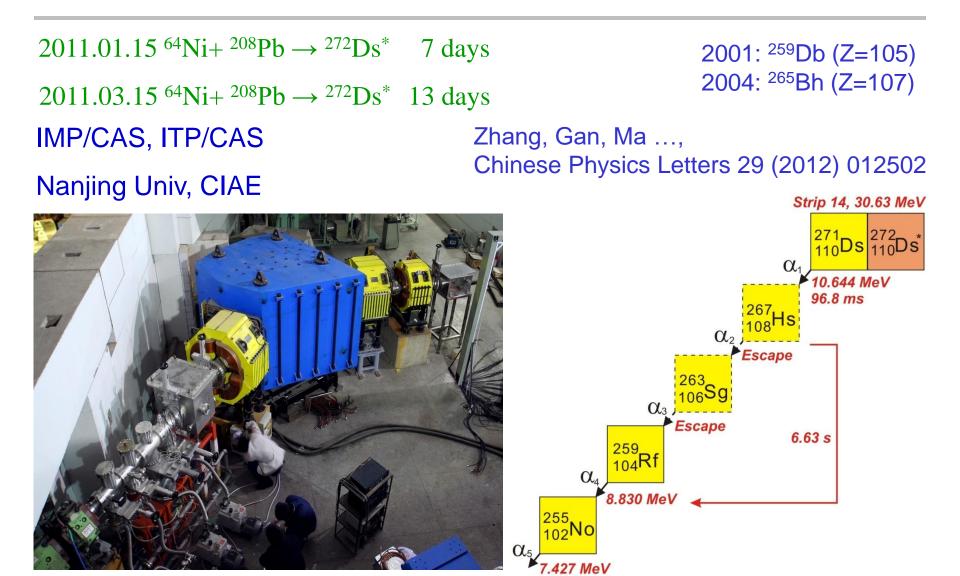


Chinese names of new elements



SGZ, <u>Nuclear Data News 118 (2017) 12</u>

HIRFL China: ²⁷¹Ds (Z = 110)



28.18 s

Courtesy of Zai-Guo Gan (甘再国)

Lecture 1

□ Predictions of the island of stability of SHN

□ Experimental progress on synthesis of SHN

□ Challenges in synthesizing SHN

Challenges to synthesize SHN & new SHEs

D Elusive island of stability

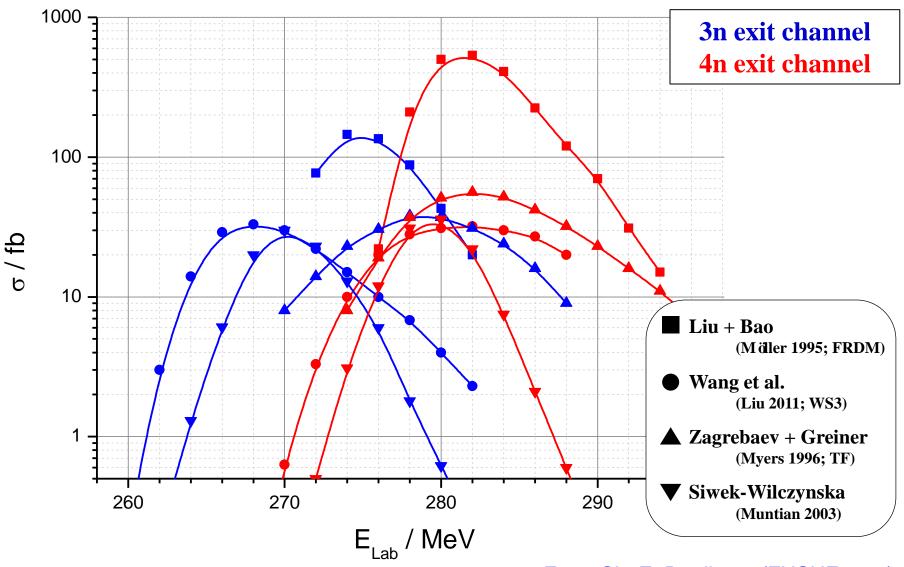
Where is the island of stability? ??? 132 = ? 114 116 120 126 138 7 **198** 228 238 258 = ? 172 176 178 184 Ν ISOTOPES OF MAP 114 island of stability proton number spherical SEA OF INSTABILITY Solfmanis shoal of 108 deformed nuclei U Th 90 mountains 126 142 162 146 184 neutron number

Challenges to synthesize SHN & new SHEs

D Elusive island of stability

□ Tiny cross sections w/ huge uncertainties $≥^{271}$ Ds: σ ~ 10 pb; HIRFL: 1 event in 20 days $≥^{278}$ Nh: σ ~ 0.02 pb; RIKEN: 3 events in 553 days

Uncertainties in predicted xsections for Z = 119



From Ch. E. Duellman (FUSHE2012)

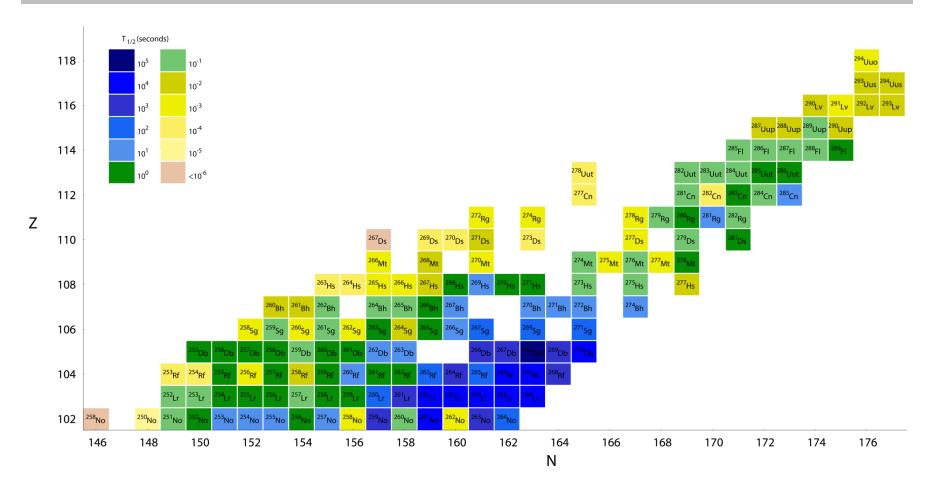
Challenges to synthesize SHN & new SHEs

Elusive island of stability

□ Tiny cross sections w/ huge uncertainties $>^{271}$ Ds: σ ~ 10 pb; HIRFL: 1 event in 20 days $>^{278}$ Nh: σ ~ 0.02 pb; RIKEN: 3 events in 553 days

D Only neutron-deficient SHN, far away from N = 184

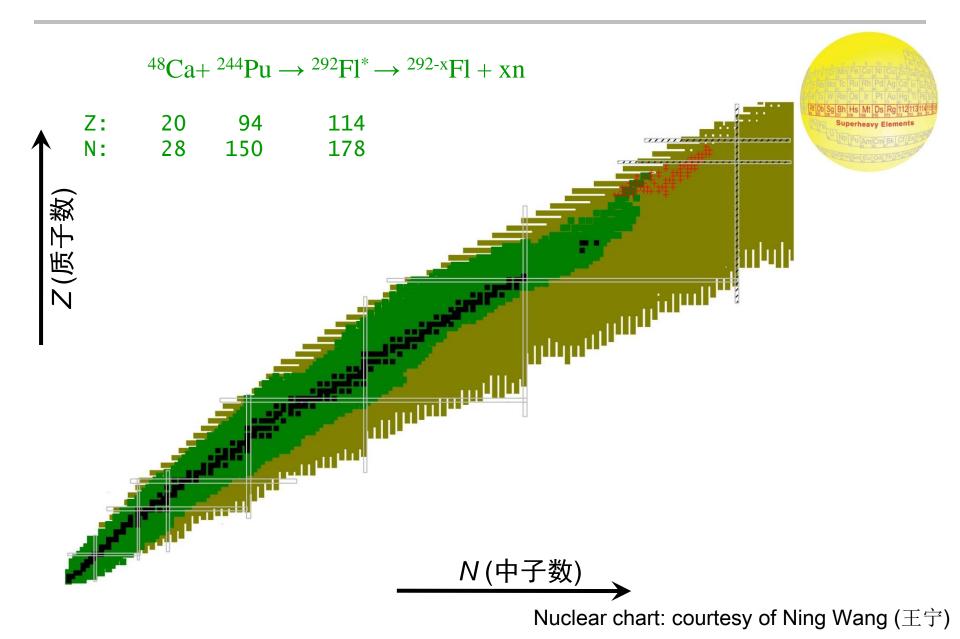
Only neutron deficient SHN produced via HI fusion



Life-time longer w/ increasing N!

Courtesy of Kai Wen (温凯)

Only neutron deficient SHN produced via HI fusion



Lecture 2

□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

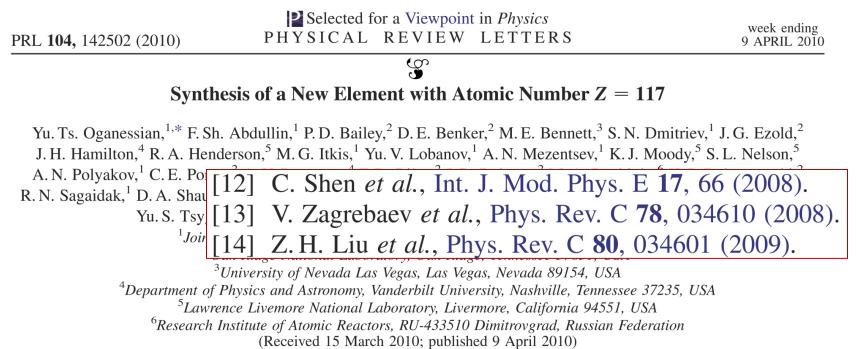
- ➤Nuclear models
- Next shell closures beyond ²⁰⁸Pb as seen from single particle spectra, shell correction energy & nuclear shapes
- ➢ Exotic shapes in SHN
- Low-lying spectra of SHN & magicities
- □ Theoretical study of decay of SHN

Study of SHN: Theory & experiment

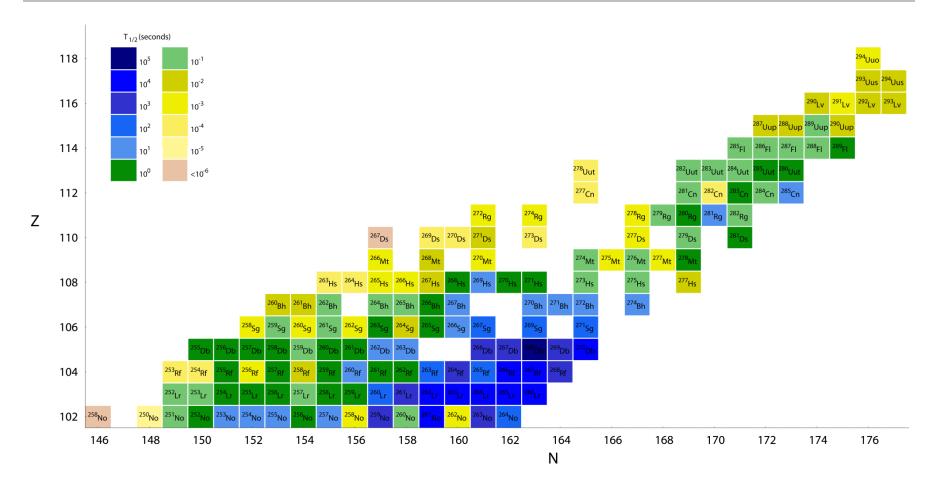
 Theory and experiment has progressed hand in hand
 1960s, heavy ion accelerators & detectors built for SHN following theoretical predictions; 1980s, important progress made in experiments

≻…

>2010, element with Z = 117



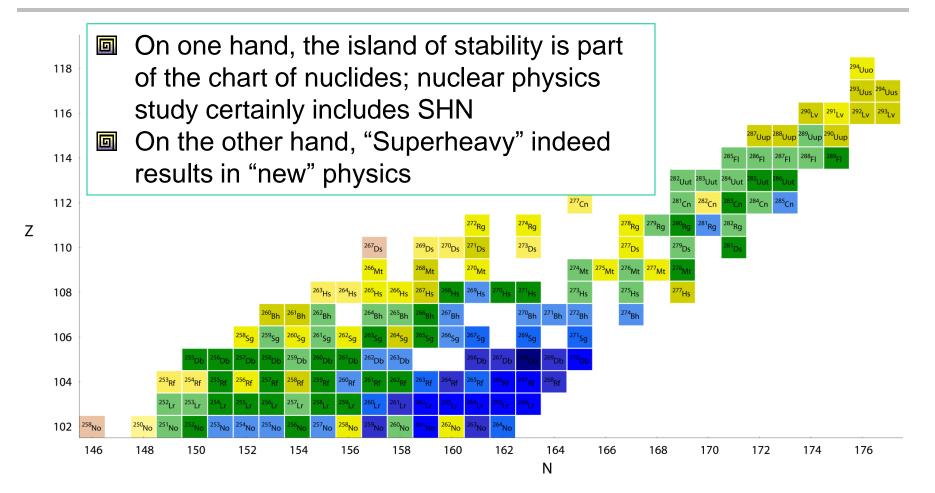
Is there new physics on the island of stability?



SHE with $Z \le 118$ have been synthesized & named Recent naming for elements with Z = 113, 115, 117 & 118

Courtesy of Kai Wen (温凯)

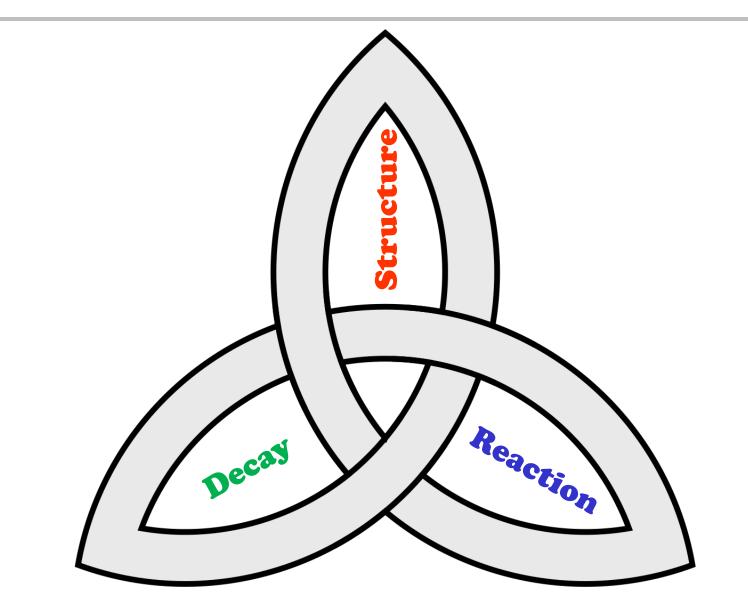
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Courtesy of Kai Wen (温凯)

Trinity in nucl. phys.: Structure, decay & reaction



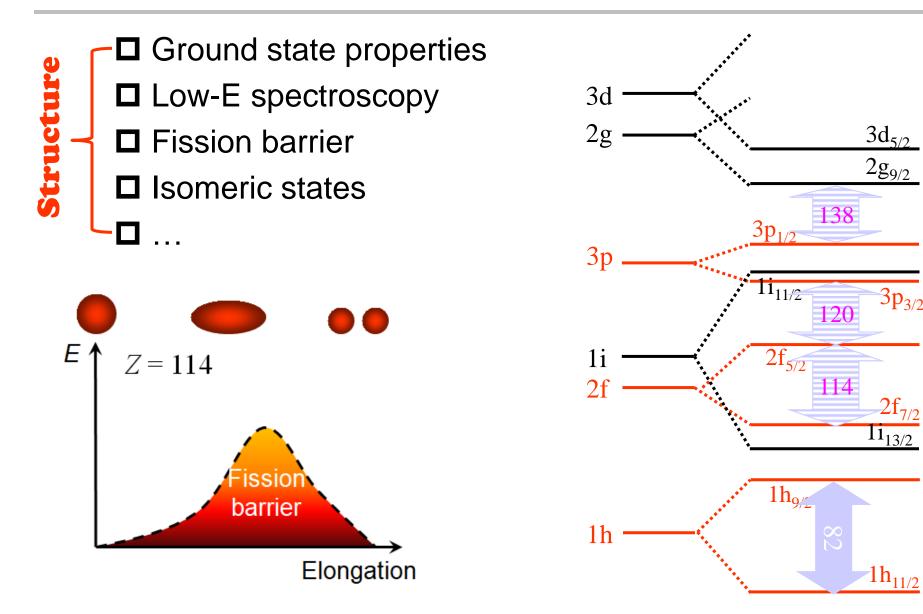
Lecture 2

□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

- ➤Nuclear models
- Next shell closures beyond ²⁰⁸Pb as seen from single particle spectra, shell correction energy & nuclear shapes
- ➤Exotic shapes in SHN
- Low-lying spectra of SHN & magicities
- □ Theoretical study of decay of SHN

Structure of SHN



3p_{3/2}

 $2f_{7/2}$

Lecture 2

□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

- Nuclear models
- Next shell closures beyond ²⁰⁸Pb as seen from single particle spectra, shell correction energy & nuclear shapes
- ➤Exotic shapes in SHN
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Nuclear models

Macroscopic models

Global behavior & average effects

□ Macroscopic-microscopic models

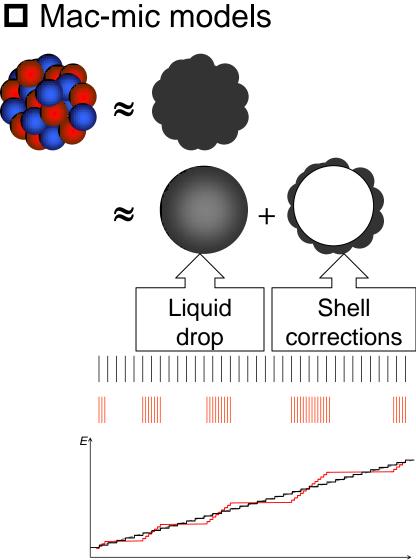
- Global behavior & average effects
- Correction from single particle motion on whole nucleus

Microscopic models

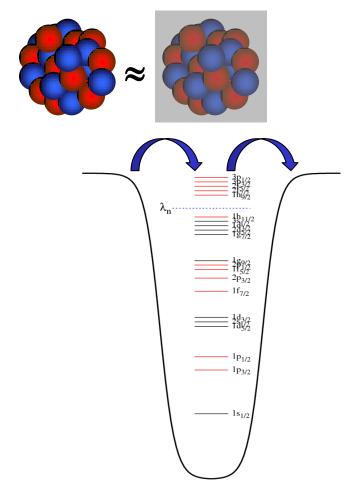
- Starting from nucleons; correlations among nucleons
- >NN interaction, mean-field approximation, residual interaction

Hu & Zhong, Macroscopic models for nuclei, 1998 (in Chinese) Greiner & Maruhn, Nuclear models, Springer, 1995 Ring & Schuck, The Nuclear Many–Body Problem, Springer, 1980

Nuclear models



Self-consistent models



Α

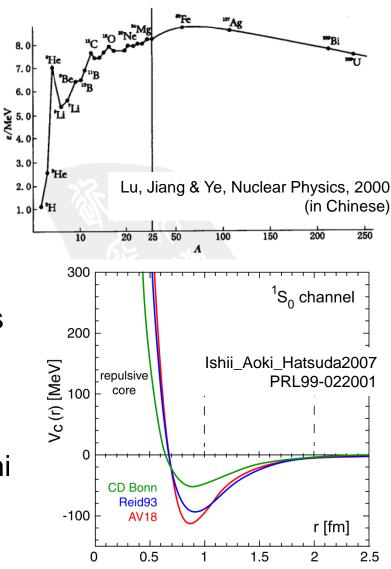
Liquid drop model: Nucleus ~ charged LD

□ Similarities

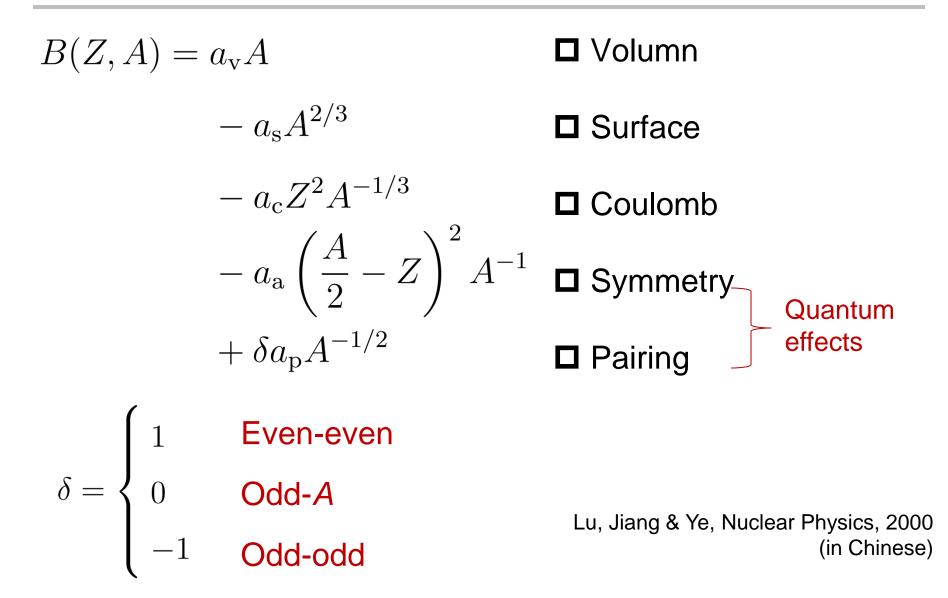
- $> B/A \sim 8 \text{ MeV} \Rightarrow$ Saturation of nuclear force
- $R \sim A^{1/3} \Rightarrow$ Incompressibility of nuclear matter

Differences

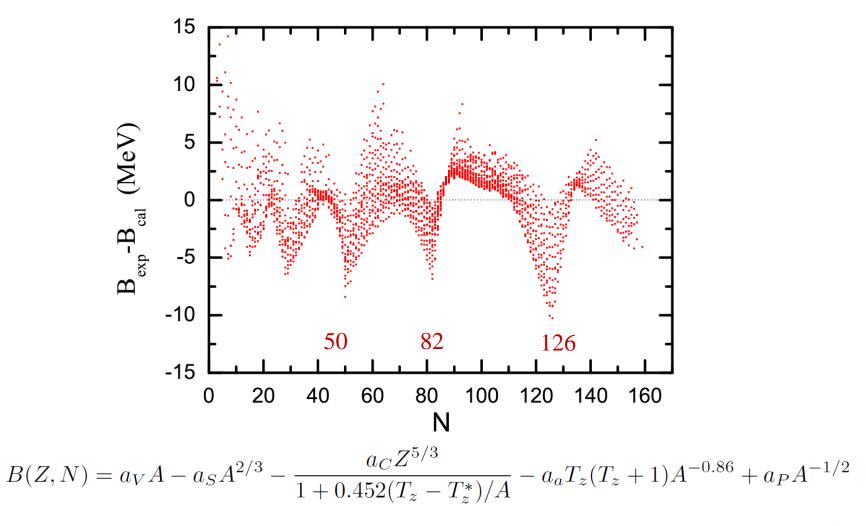
- Mean distance between nucleons larger than that corresponding to the minimum in NN potential
- Mean free path of nucleon comparable to nuclear size (Fermi gas model)



Liquid drop model: Nucleus ~ charged LD



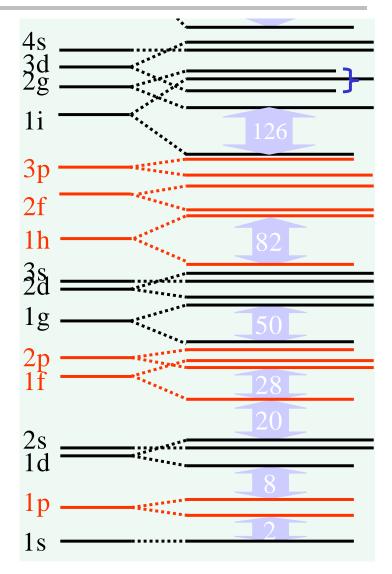
Comparison w/ experiment



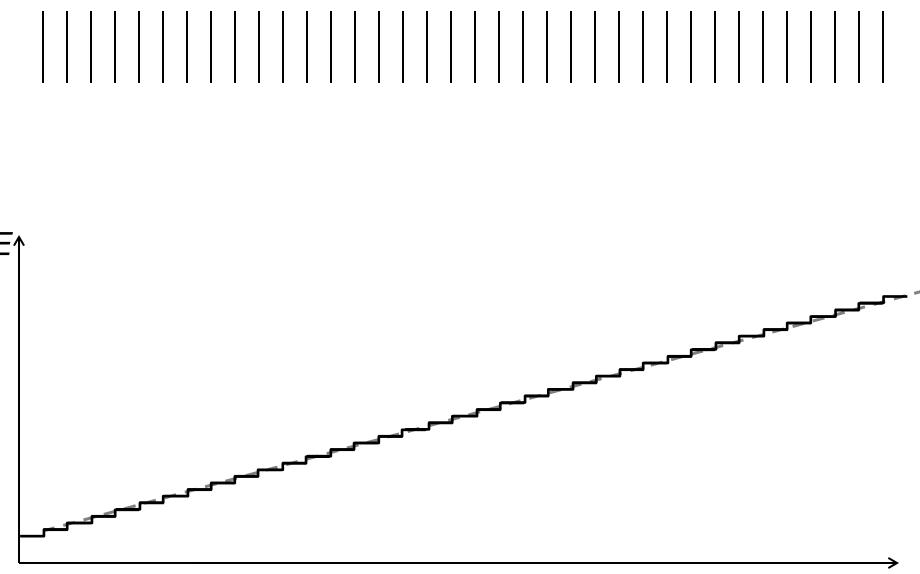
Courtesy of Yu-Chen Cao (曹宇晨)

Single nucleon potential, spectra & magicities

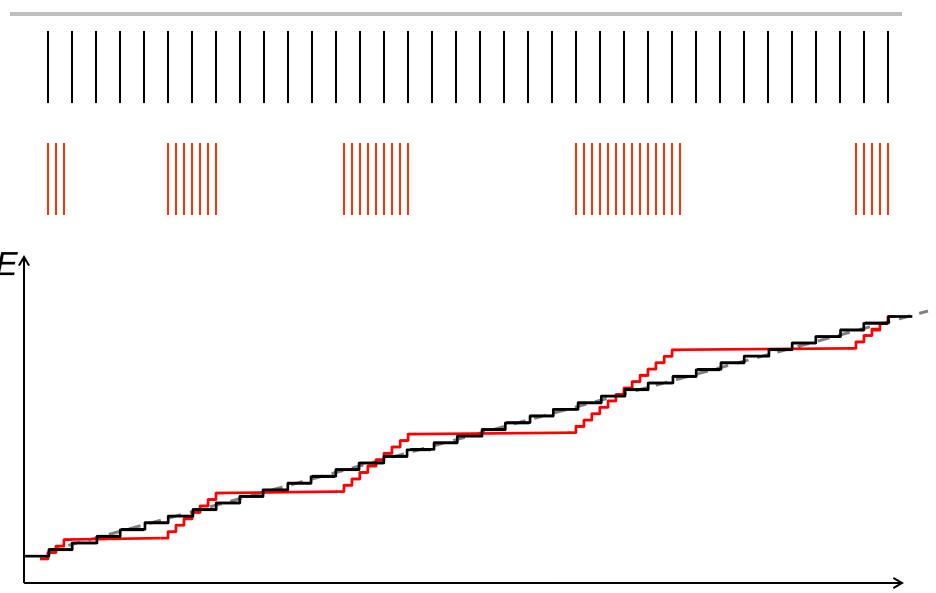
- □ Harmonic oscillator & square well
- Woods-Saxon
- □ Self-consistent (self-bound)



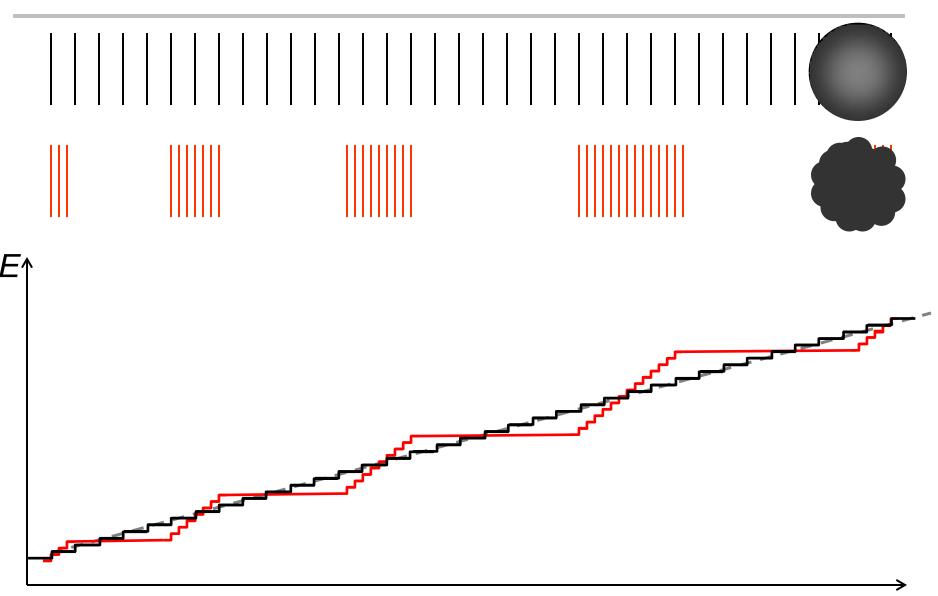
Quantum shell effects



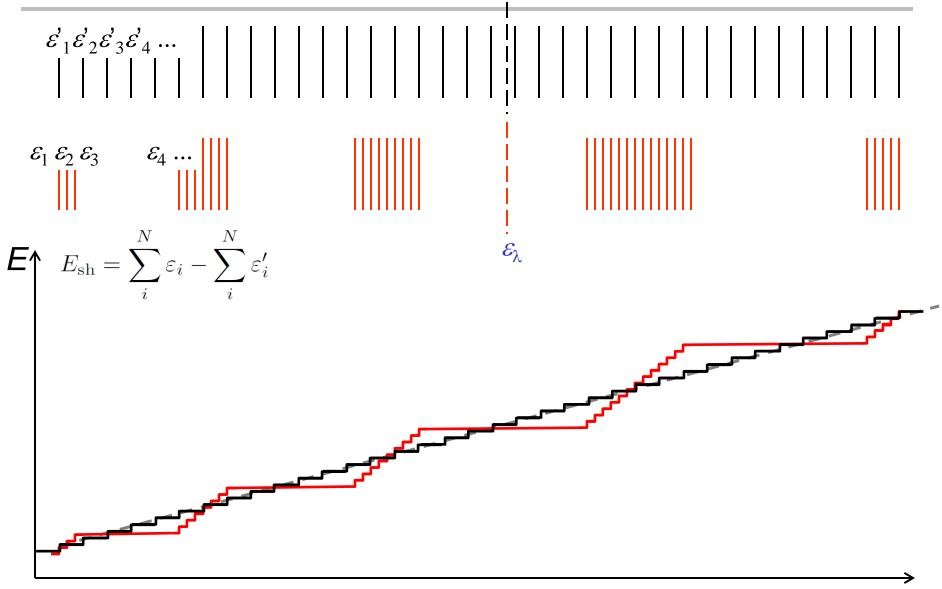
Quantum shell effects



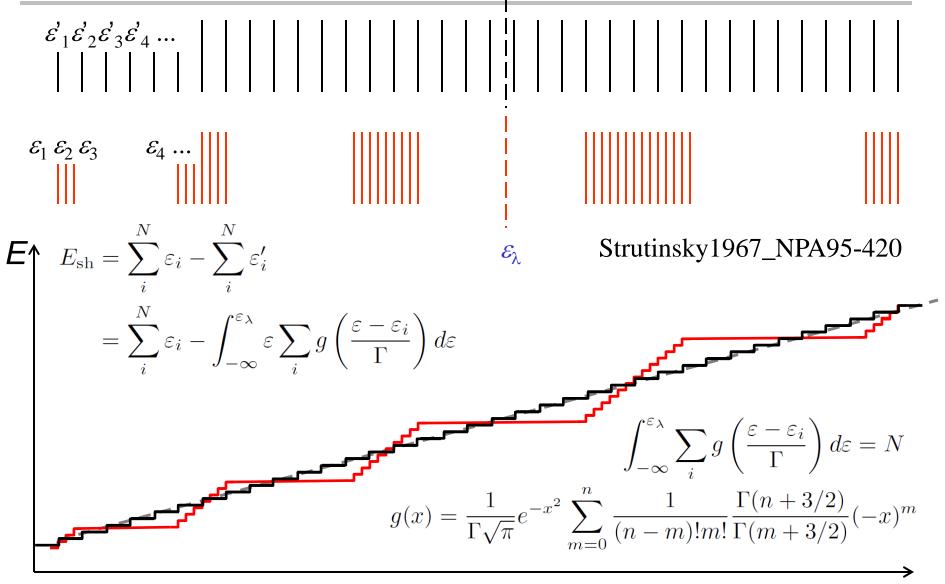
Quantum shell effects



Microscopic shell corrections

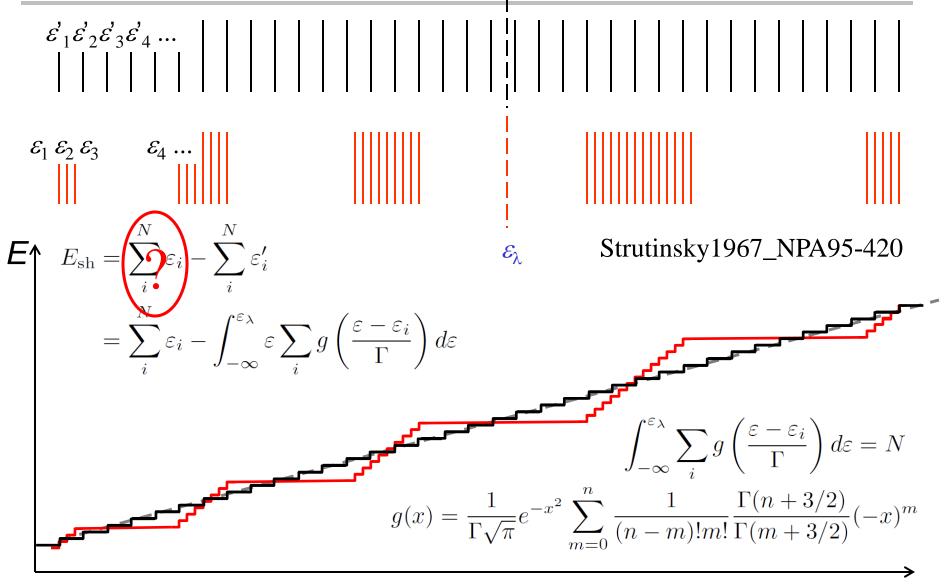


Shell corrections: Strutinsky mothod



Α

Shell corrections: Strutinsky mothod

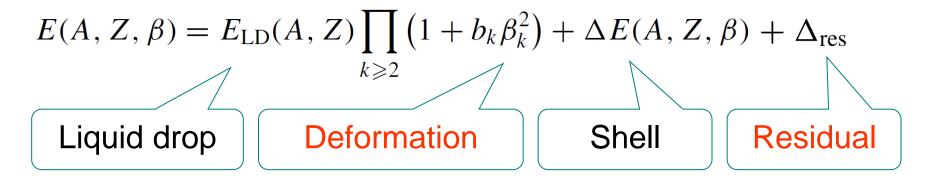


Α

Finite range droplet model (FRDM)

 $E_{\rm mac}(Z, N, {\rm shape}) =$ Moeller+2016 ADNDT109-110-1 $M_{\rm H}Z + M_{\rm n}N$ mass excesses of Z hydrogen atoms and N neutrons + $\left(-a_1 + J\overline{\delta}^2 - \frac{1}{2}K\overline{\varepsilon}^2\right)A$ volume energy $-c_a(N-Z)$ charge-asymmetry energy + $\left(a_2B_1 + \frac{9}{4}\frac{J^2}{Q}\overline{\delta}^2\frac{B_s^2}{B_1}\right)A^{2/3}$ surface energy $+ W \left(|I| + \begin{cases} 1/A, & Z \text{ and } N \text{ odd and equal} \\ 0, & \text{otherwise} \end{cases} \right)$ $+a_3A^{1/3}B_k$ curvature energy Wigner energy $+ a_0 A^0 A^0$ energy $+\begin{cases} +\overline{\Delta}_{p} + \overline{\Delta}_{n} - \delta_{np}, & Z \text{ and } N \text{ odd} \\ +\overline{\Delta}_{p}, & Z \text{ odd and } N \text{ even} \\ +\overline{\Delta}_{n}, & Z \text{ even and } N \text{ odd} \\ +0, & Z \text{ and } N \text{ even} \end{cases}$ $+ c_1 \frac{Z^2}{A^{1/3}} B_3$ Coulomb energy $-c_2 Z^2 A^{1/3} B_r$ volume redistribution energy $-c_4 \frac{Z^{4/3}}{A^{1/3}}$ Coulomb exchange correction average pairing energy $-c_5 Z^2 \frac{B_W B_s}{B_s}$ surface redistribution energy $-a_{\rm el}Z^{2.39}$ energy of bound electrons $+f_0 \frac{Z^2}{\Lambda}$ proton form-factor correction to the Coulomb energy

Weizsäcker-Skyrme (WS) mass model



 $E_{\rm LD}(A, Z) = a_v A + a_s A^{2/3} + E_C + a_{\rm sym} I^2 A + a_{\rm pair} A^{-1/3} \delta_{np}$

 $\Delta E(A, Z, \beta)$: Shell corrections Δ_{res} (residual): Mirror, pairing, Wigner corrections, ...

Macro-micro concept & Skyrme energy density functional

N. Wang, M. Liu, et al., PRC81-044322; PRC82-044304; PRC84-014333

Covariant Density Functional Theory (CDFT)

$$\mathcal{L} = \bar{\psi}_{i} \left(i\partial - M \right) \psi_{i} + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) - g_{\sigma} \bar{\psi}_{i} \sigma \psi_{i} - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega^{\mu} - g_{\omega} \bar{\psi}_{i} \phi \psi_{i} - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_{\rho}^{2} \vec{\rho}_{\mu} \vec{\rho}^{\mu} - g_{\rho} \bar{\psi}_{i} \vec{\rho} \vec{\tau} \psi_{i}$$
 Serot_Walecka1986_ANP16-1
 - $\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_{i} \frac{1 - \tau_{3}}{2} \mathcal{A} \psi_{i},$ Reinhard1989 RP52-439
 - $\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_{i} \frac{1 - \tau_{3}}{2} \mathcal{A} \psi_{i},$ Ring1996_PPNP37-193
 Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101
 Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470
 ($\boldsymbol{\alpha} \cdot \mathbf{p} + \beta (M + S(\mathbf{r})) + V(\mathbf{r})) \psi_{i} = \epsilon_{i} \psi_{i}$ Liang_Meng_SGZ2015_PR570-1
 $\left(-\nabla^{2} + m_{\sigma}^{2} \right) \sigma = -g_{\sigma} \rho_{S} - g_{2} \sigma^{2} - g_{3} \sigma^{3}$ ($-\nabla^{2} + m_{\omega}^{2} \right) \omega = g_{\omega} \rho_{V} - c_{3} \omega^{3}$

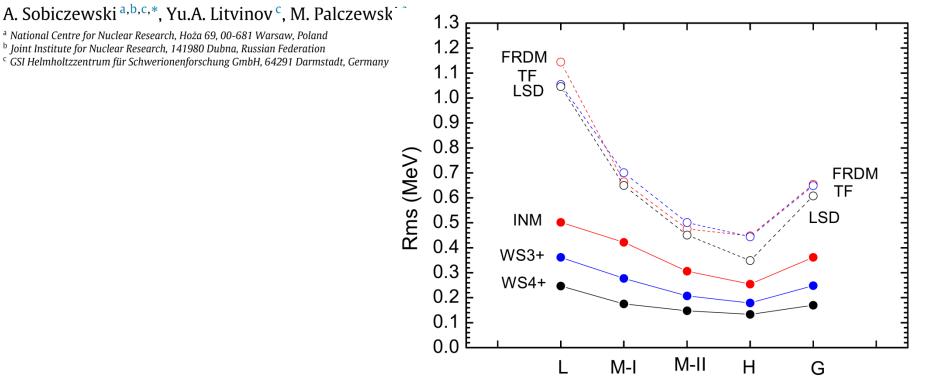
 $-\nabla^2 A = e\rho_C$

Various nuclear mass models: Comparison

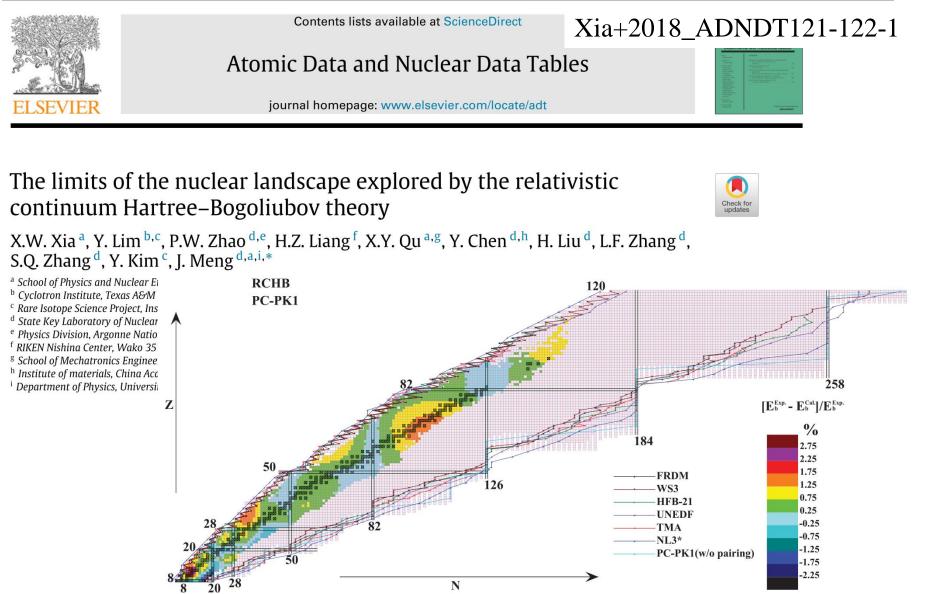


Detailed illustration of the accuracy of currently used nuclear-mass models





Relativistic continuum Hartree-Bogoliubov (RCHB) theory

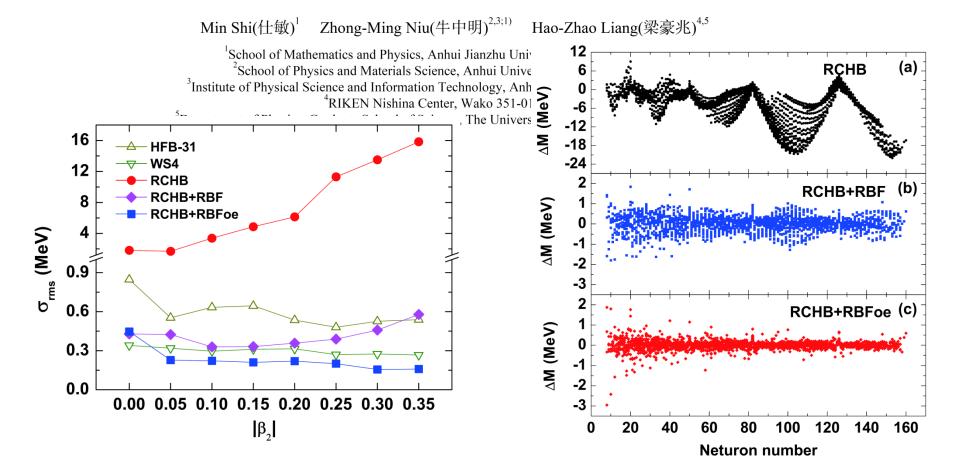


RCHB + radial basis function approach

Chinese Physics C Vol. 43, No. 7 (2019) 074104

Shi_Niu_Liang2019_ChinPhysC43-074104

Mass predictions of the relativistic continuum Hartree-Bogoliubov model with radial basis function approach*



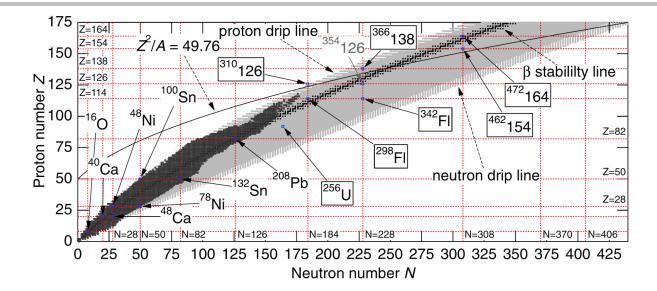
Lecture 2

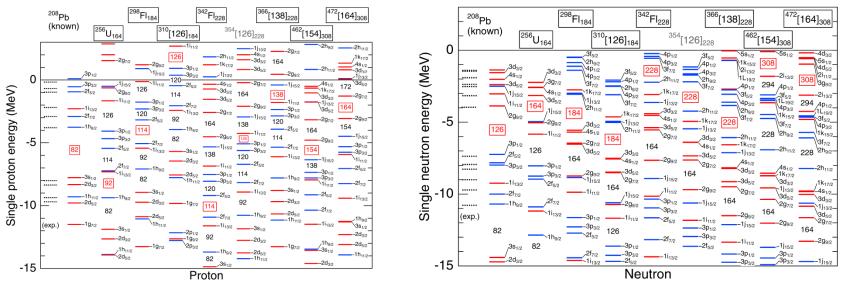
□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

- ➤Nuclear models
- Next shell closures beyond ²⁰⁸Pb as seen from single particle spectra, shell correction energy & nuclear shapes
- ➤Exotic shapes in SHN
- Low-lying spectra of SHN & magicities
- □ Theoretical study of decay of SHN

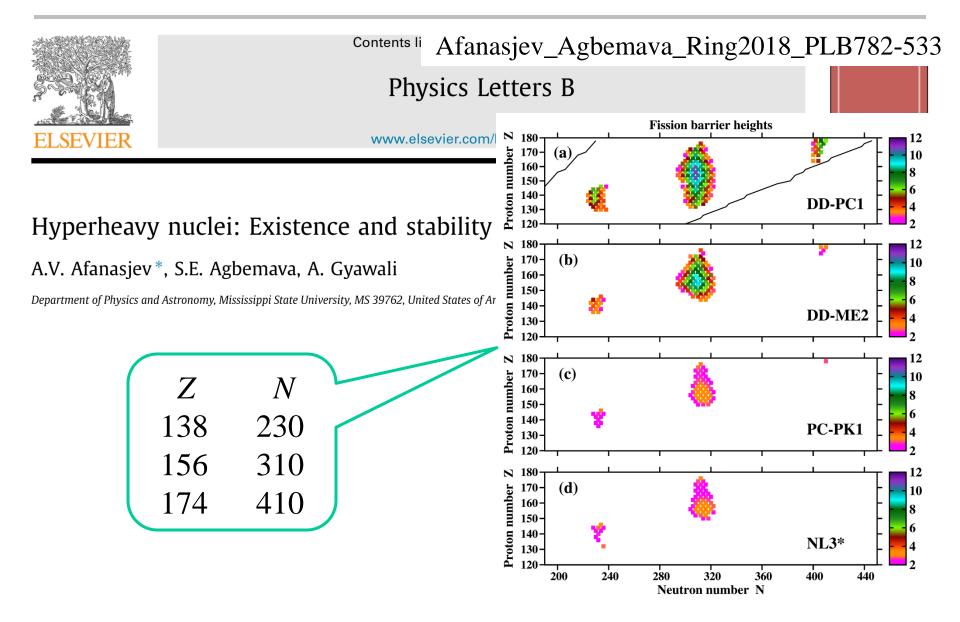
Modified Woods-Saxon potential





Koura&Chiba2013_JPSJ82-014201

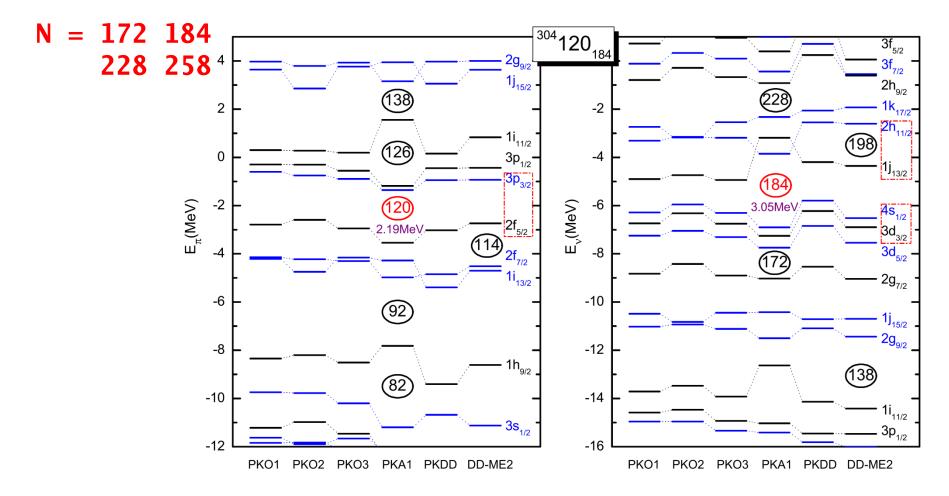
Hyperheavy nuclei



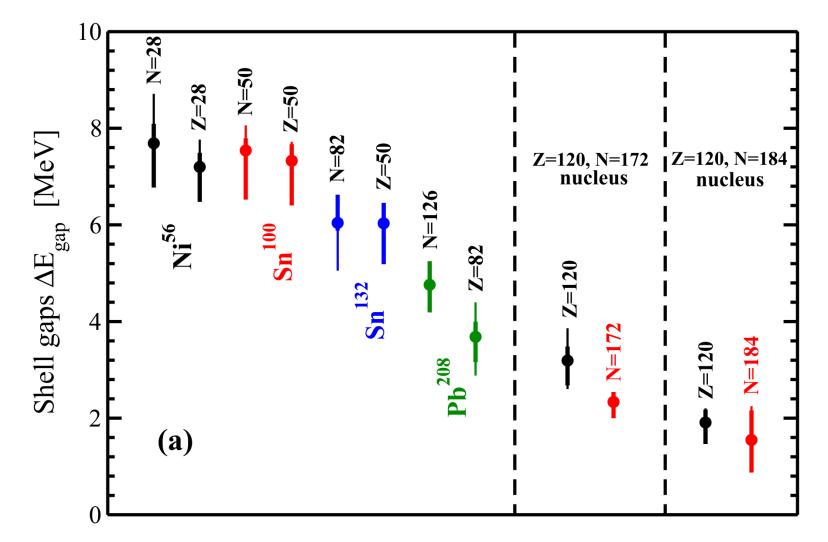
Relativistic Hartree-Fock-Bogoliubov model



Li_Long_Margueron_Giai2014_PLB732-169

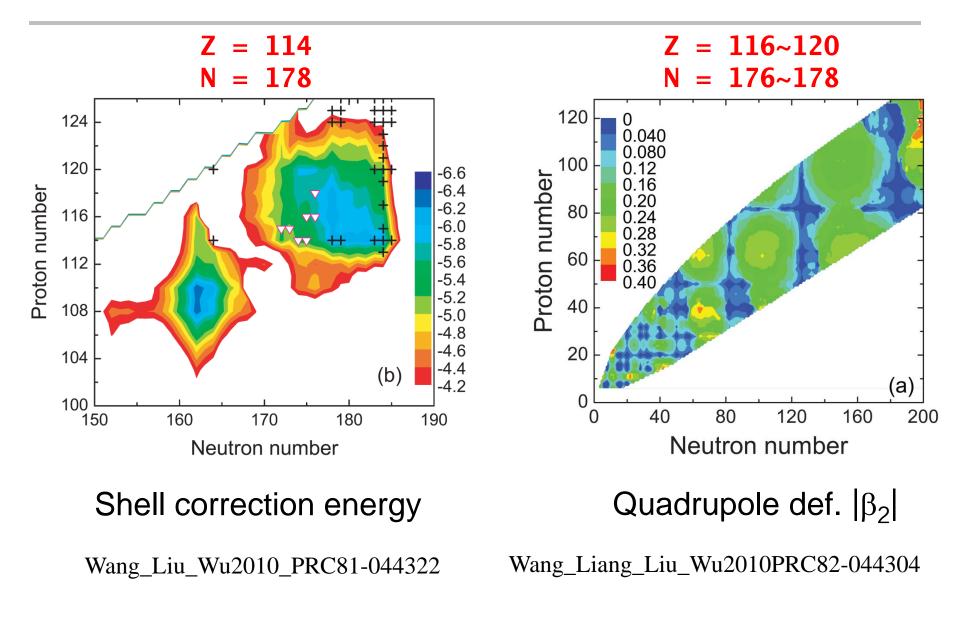


Single-particle level densities & gaps



Agbemava_Afanasjev_Nakatsukasa&Ring2015_PRC92-054310

Microscopic correction energies & deformations



Where is the island of stability? ??? 132 = ? 114 116 120 126 138 7 **198** 228 238 258 = ? 172 176 178 184 Ν ISOTOPES OF MAP 114 island of stability proton number spherical SEA OF INSTABILITY Solfmanis shoal of 108 deformed nuclei U Th 90 mountains 126 142 162 146 184 neutron number

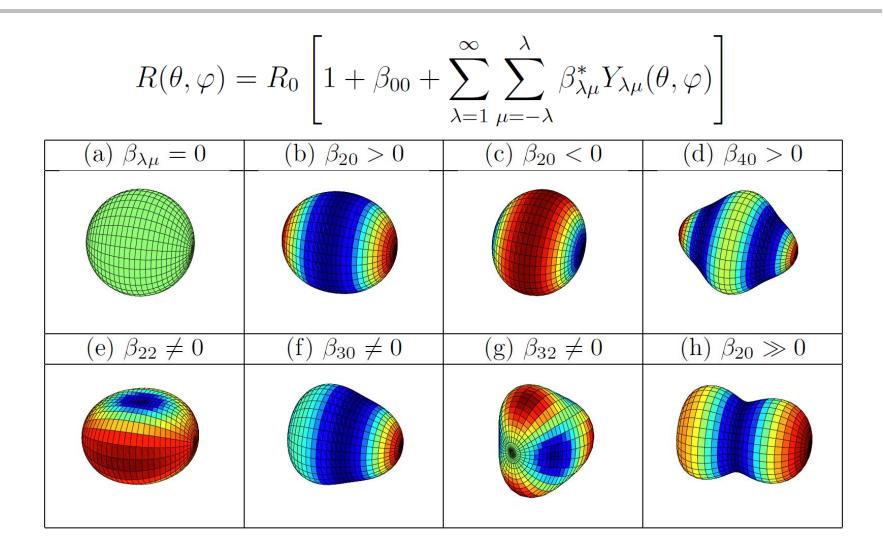
Lecture 2

□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

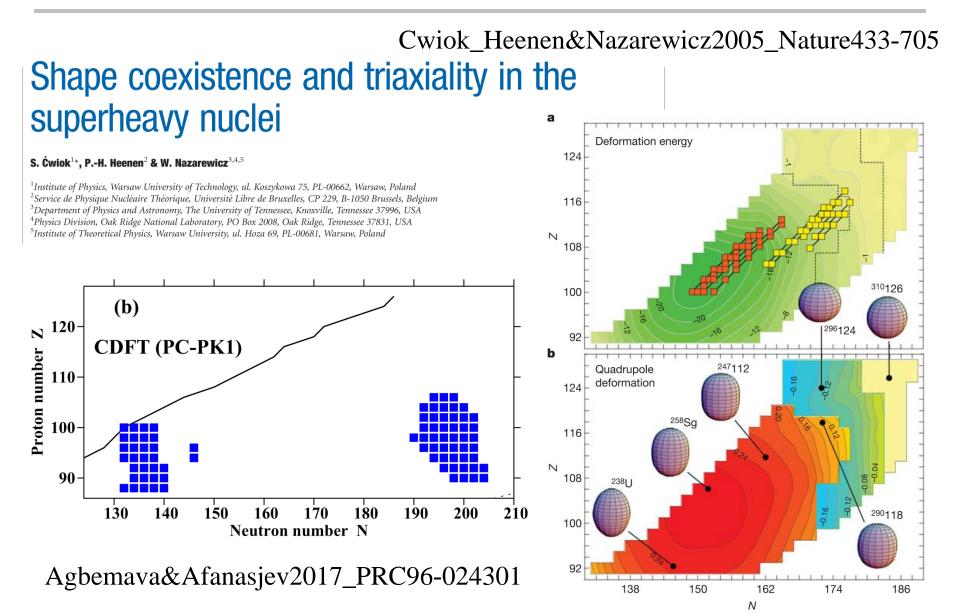
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Nuclear shapes

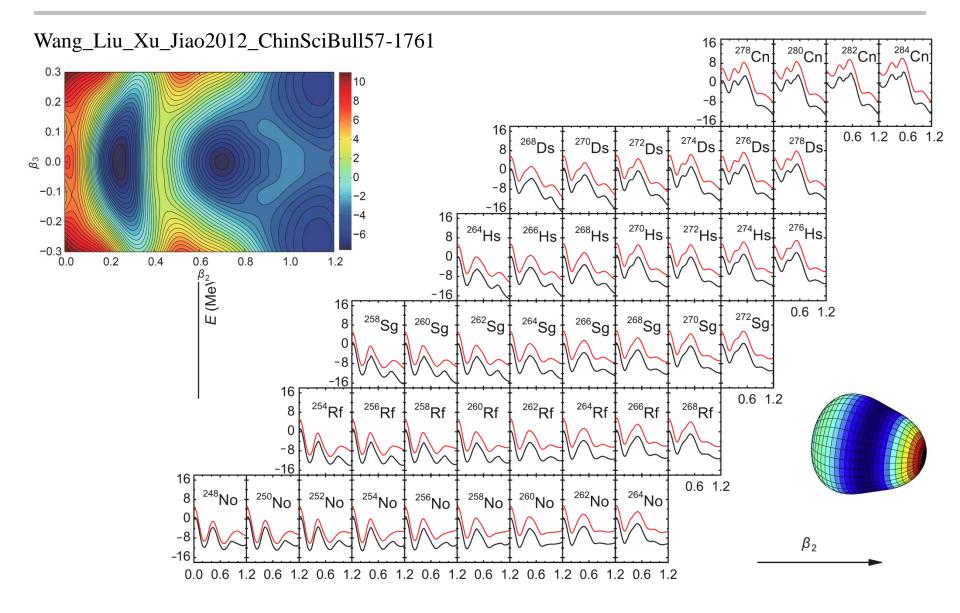


Courtesy of Bing-Nan Lu (吕炳楠)

Shapes of SHN: Triaxial & octupole



Octupole correlations in SHN



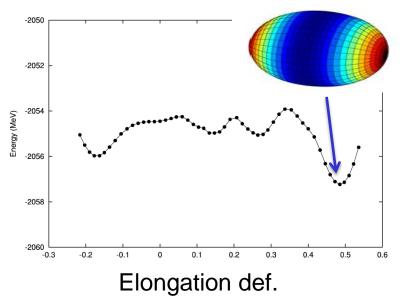
Possible exotic shapes in SHN

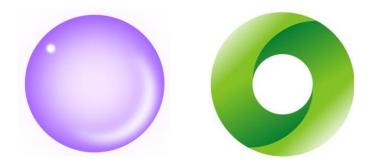
Bubble or toroidal

1960s, H. A. Bethe, J. A. Wheeler, ... Wong1973_APNY77-279 Dietrich_Pomorski1998_PRL80-37 Pei_Xu_Stevenson 2005_PRC71-034302

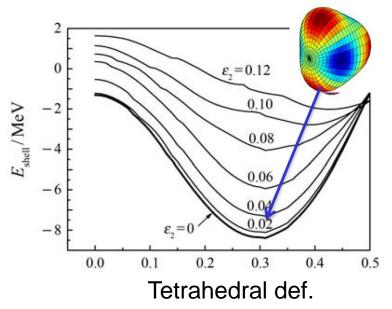
Superdeformed or tetrahedral

Ren_Toki2001_NPA689-691 Ren2002_PRC65-051304R





Chen_Gao2010_NPA834-380c 2013_NPR30-278



Lecture 2

□ Challenges in synthesizing SHN

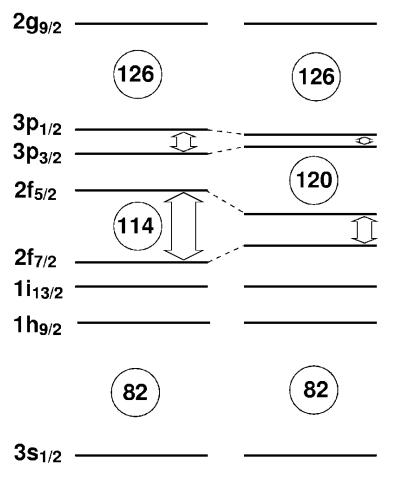
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Spectroscopy of nuclei with Z~100

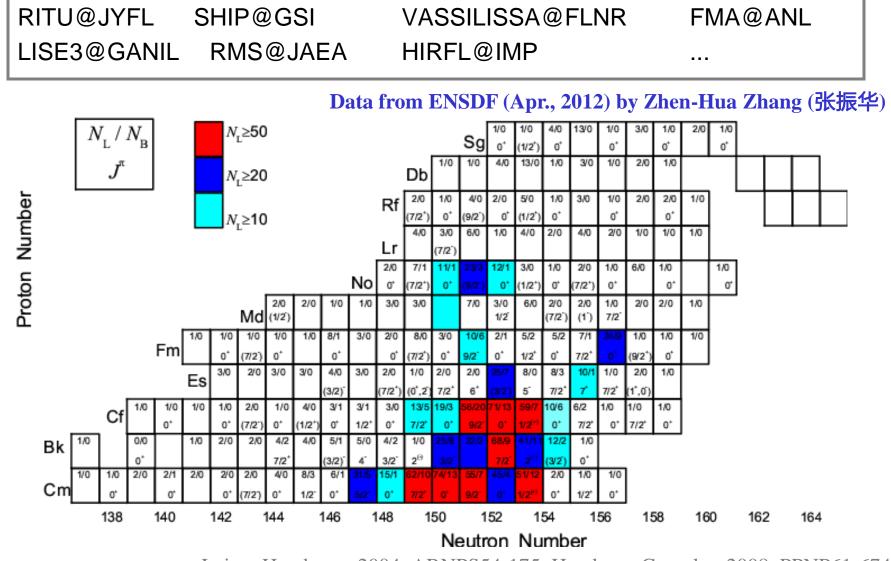
Synthesis of SHN \Rightarrow Decay modes & energies; X-sections, ...

- Spectroscopy of SHN
 Detailed structure & stability
- Spectroscopy of deformed nuclei with Z ~ 100 & N ~ 152
 - Of interest in itself --- occurrence of deformation & K-isomerism
 - Orbitals around the Fermi level in these nuclei stem from those connected to the spherical shell gaps in SHN (1/2⁻[521])



Herzberg_Greenlees_Butler... 2006_Nature442-896

Experimental facilities & status



Leino_Hessberger2004_ARNPS54-175; Herzberg_Greenlees2008_PPNP61-674

Theoretical study of low-lying spectra

Self-consistent approaches

Egido_Robledo2000_PRL85-1198 Delaroche...2006_NPA771-103 Adamian...2011_PRC84-024324 Afanasjev...2003_PRC67-024309 Bender...2003_NPA723-354

Macroscopic-Microscopic models

Cwiok...1994_NPA573-356 Muntian...1999_PRC60-041302R Sobiczewski...2001_PRC63-034306 Parkhomenko Sobiczewski2004 APPB35-2447 Parkhomenko_Sobiczewski2005_APPB36-3115

Adamian...2011_PRC 84-024324

Projected shell model

Sun...2008_PRC77-044307

Al-Khudair...2009_PRC79-034320

Cranking shell model

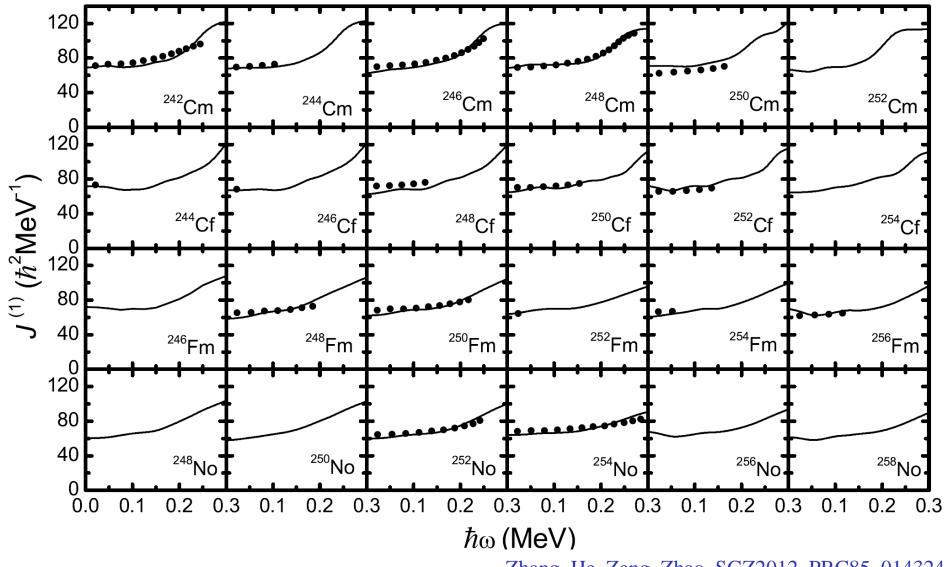
He...2009_NPA817-45 Liu...2012_PRC86-011301R Zhang...2011_PRC83-011304R Zhang...2012_PRC85_014324 Zhang...2013_PRC87-054308

Chen...2008 PRC77-061305

Cranked Nilsson model w/ pairing treated by a particle number conserving method

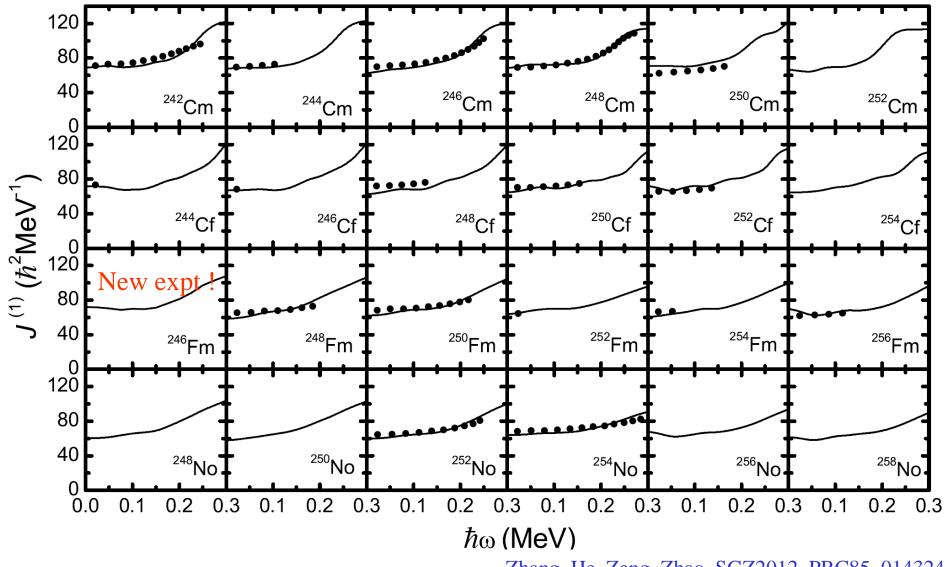
$$\begin{split} H_{\text{CSM}} &= H_0 + H_{\text{P}} = H_{\text{Nil}} - \omega J_x + H_{\text{P}} \\ H_{\text{Nil}} &= \frac{1}{2} \hbar \omega_0(\varepsilon_2, \varepsilon_4) \left[-\nabla_{\rho}^2 + \frac{1}{3} \left(2 \frac{\partial^2}{\partial \zeta^2} - \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \eta^2} \right) + \rho^2 \\ &- \frac{2}{3} \varepsilon_2 \rho^2 P_2(\cos \theta_t) + 2\varepsilon_4 \rho^2 P_4(\cos \theta_t) \right] & \text{Zhang...2011_PRC83-011304R} \\ &- 2\kappa \hbar \mathring{\omega}_0 \left(\vec{s} \cdot \vec{l}_t - \mu \left(\rho^4 - \langle \rho^4 \rangle_N \right) \right) , & \text{Zhang...2012_PRC85_014324} \\ &- 2\kappa \hbar \mathring{\omega}_0 \left(\vec{s} \cdot \vec{l}_t - \mu \left(\rho^4 - \langle \rho^4 \rangle_N \right) \right) , & \text{Zhang} ...2012_PRC85_014324} \\ H_{\text{P}}(0) &= -G_0 \sum_{\xi\eta} a_{\xi}^{\dagger} a_{\bar{\xi}}^{\dagger} a_{\bar{\eta}} a_{\eta} & H_{\text{P}}(2) = -G_2 \sum_{\xi\eta} q_2(\xi) q_2(\eta) a_{\xi}^{\dagger} a_{\bar{\xi}}^{\dagger} a_{\bar{\eta}} a_{\eta} \\ \hline \frac{N l \kappa_{\rho} \mu_{\rho} N l \kappa_{n} \mu_{n}}{4 0.2,4 0.0670 0.654} \\ 5 1 0.0250 0.710 6 0 0.1600 0.320 \\ 3 0.0570 0.800 2 0.0640 0.200 \\ 5 0.0570 0.710 4,6 0.0680 0.260 \\ 6 0,2,4,6 0.0570 0.654 7 1,3,5,7 0.0634 0.318 \end{split}$$

Moments of inertia of even-even nuclei



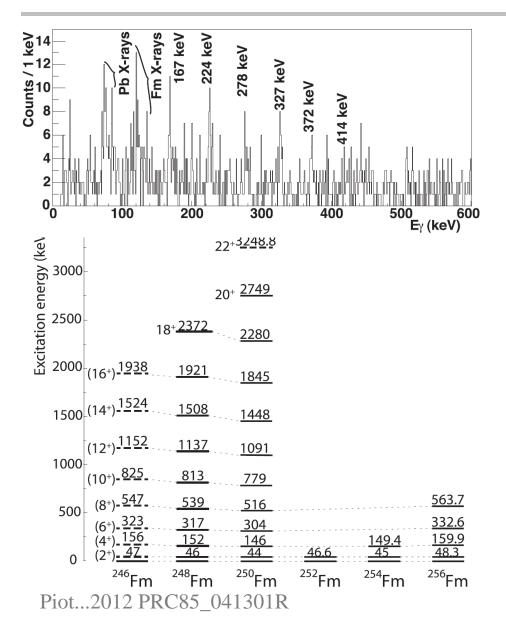
Zhang_He_Zeng_Zhao_SGZ2012_PRC85_014324

Moments of inertia of even-even nuclei

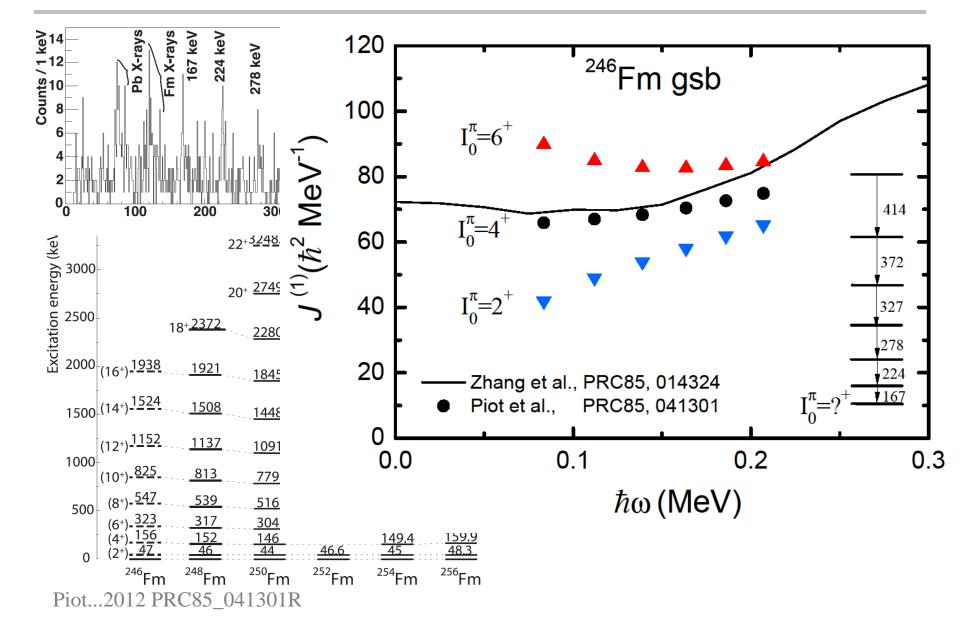


Zhang_He_Zeng_Zhao_SGZ2012_PRC85_014324

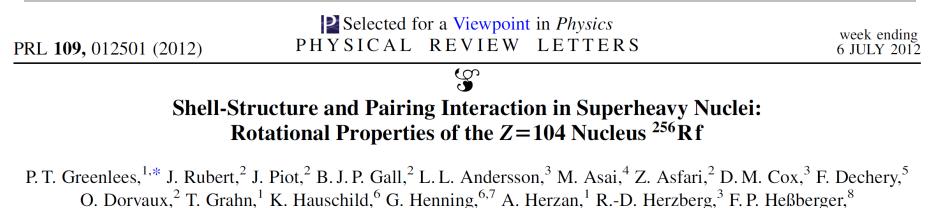
²⁴⁶Fm: ground state band observed @ Jyvaskyla



²⁴⁶Fm: ground state band observed @ Jyvaskyla



²⁵⁶Rf: ground state band observed @ Jyvaskyla



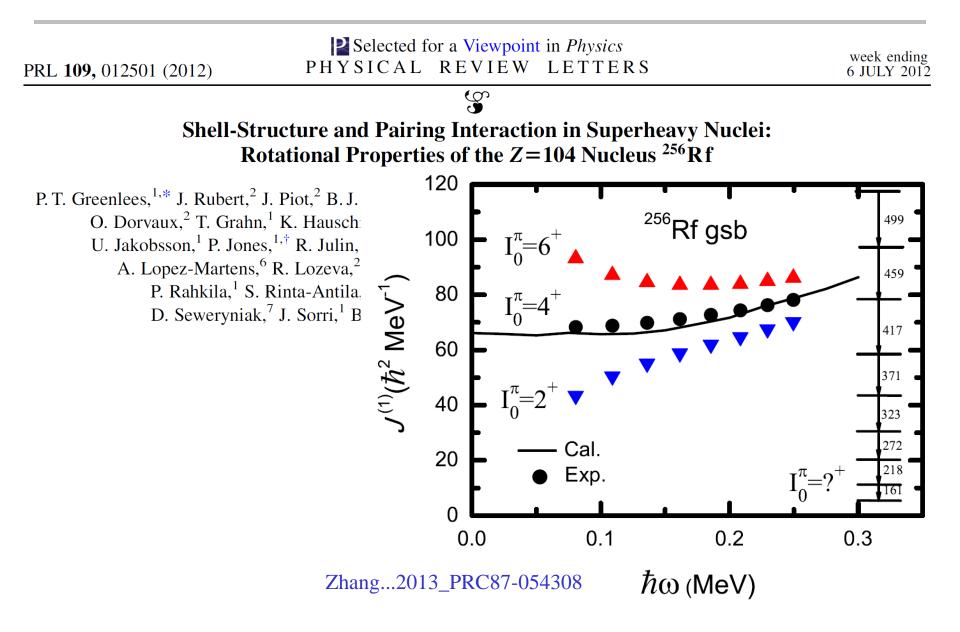
U. Jakobsson,¹ P. Jones,^{1,†} R. Julin,¹ S. Juutinen,¹ S. Ketelhut,¹ T.-L. Khoo,⁷ M. Leino,¹ J. Ljungvall,⁶

A. Lopez-Martens,⁶ R. Lozeva,² P. Nieminen,¹ J. Pakarinen,⁹ P. Papadakis,³ E. Parr,³ P. Peura,¹

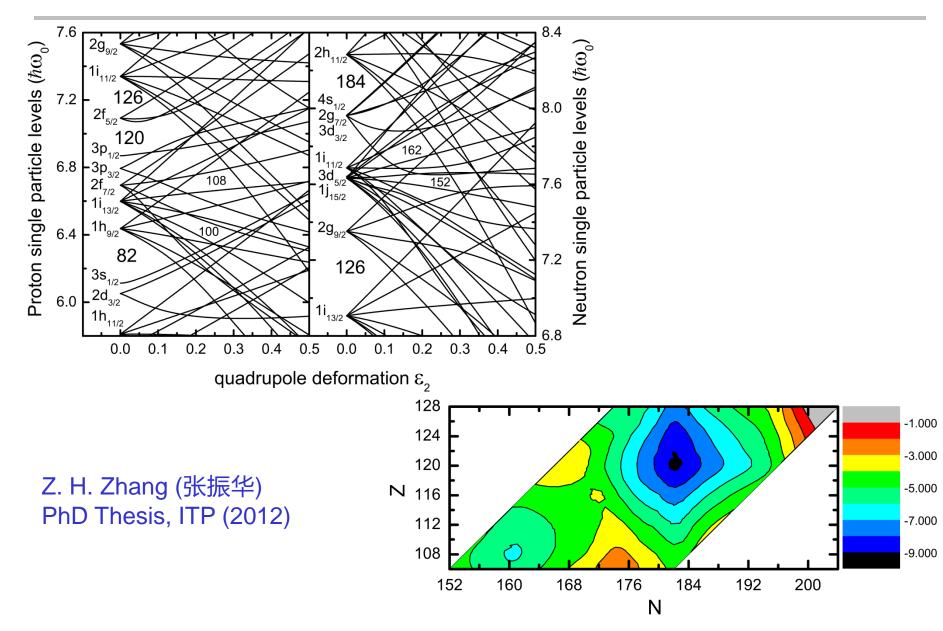
P. Rahkila,¹ S. Rinta-Antila,¹ P. Ruotsalainen,¹ M. Sandzelius,¹ J. Sarén,¹ C. Scholey,¹

D. Seweryniak,⁷ J. Sorri,¹ B. Sulignano,⁵ Ch. Theisen,⁵ J. Uusitalo,¹ and M. Venhart¹⁰

²⁵⁶Rf: ground state band observed @ Jyvaskyla



Nilsson diagrams & E_{mic}



Lecture 2

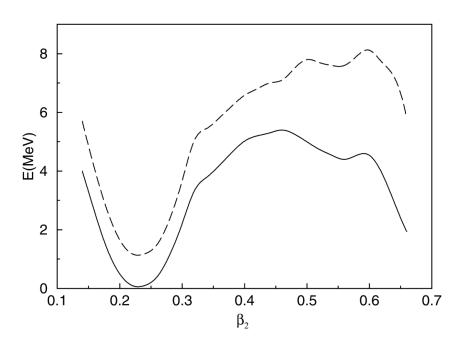
□ Challenges in synthesizing SHN

Theoretical study of structure of SHN

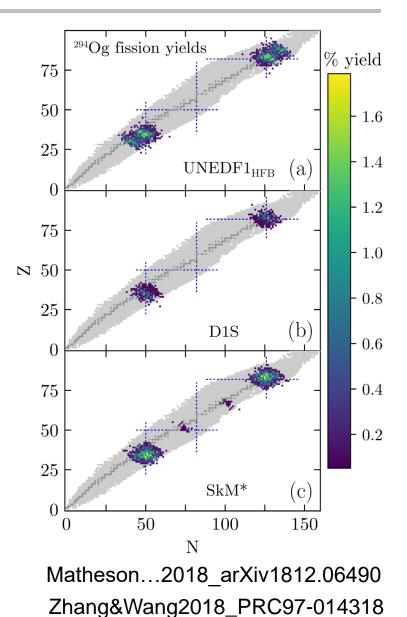
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Decay & spontaneous fission of SHN

- α, β & γ decays
 Spontaneous fission
 Long-lived isomers
- Cluster radioactivity



Xu_Zhao_Wyss&Walker2004_PRL92-252501



Lectures 3 & 4

□ Challenges in synthesizing SHN

□ Synthesis mechanism of SHN

Large uncertainties in predicted Xsections

- Heavy ion fusion reactions
 - Capture
 - Fusion
 - Survival against fission
- Multi-nucleon transfer reactions
- Reactions w/ radioactive ion beams

Lectures 3 & 4

□ Challenges in synthesizing SHN

□ Synthesis mechanism of SHN

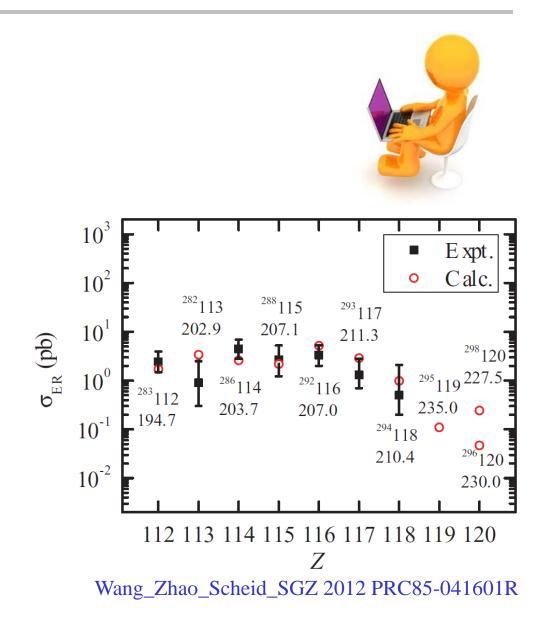
Large uncertainties in predicted Xsections

- Heavy ion fusion reactions
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 - Fusion
 - Survival against fission
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How to synthesize SHE w/ Z>118?



How to synthesize SHE w/ Z>118?

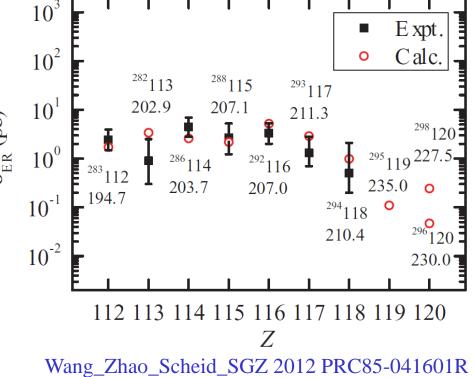


How to synthesize SHE w/ Z>118?

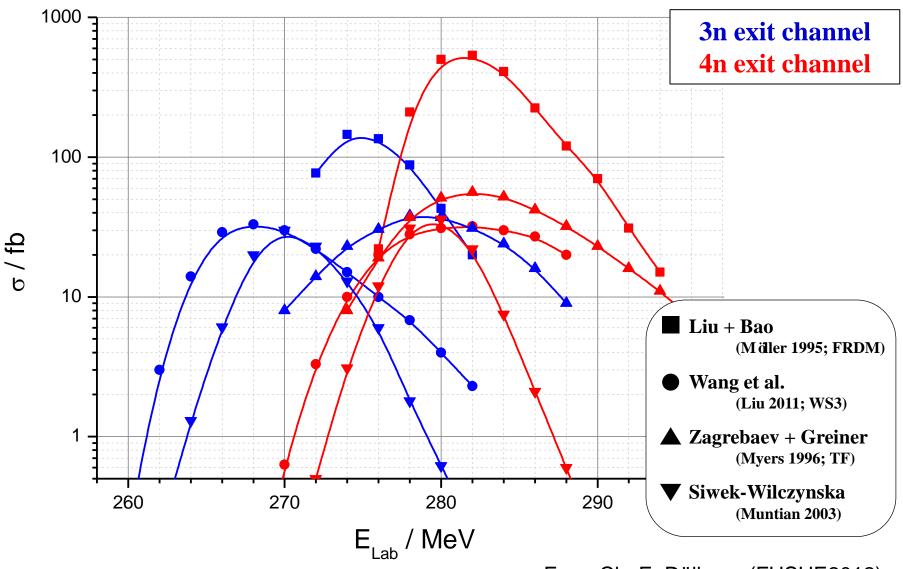
Feng Jin Li Scheid2007 PRC76-044606 Nasirov Giardina Mandaglio Manganaro Hanappe Heinz Hofmann Muminov Scheid2009 PRC79-024606 Adamian Antonenko Scheid2009 EPJA41-235 Gan Zhou Huang Feng Li2011 SciChinaPMA54S1-61 Nasirov Mandaglio Giardina Sobiczewski Muminov2011 PRC84-044612 Liu Bao2009 PRC80-054608 Siwek-Wilczynska_Cap_Wilczynski2010_IJMPE19-500 Liu Bao2011 PRC83-044613 10^{3} Liu Bao2011 PRC84-031602R Zagrebaev Greiner2008 PRC78-034610 10^{2} Wang Tian Scheid2011 PRC84-061601R Siwek-Wilczynska_Cap_Kowal_Sobiczewski_ Wilczynski 2012_PRC86-014611 10^{1} (qd) Liang Zhu Liu Wang 2012 PRC86-037602 Liu Bao2013 PRC87-034616 10^{0} $\sigma_{\rm ER}$ Zhang Wang Ren2013 NPA909-36 Zhu Xie Zhang2014 PRC89-024615 10^{-1} Bao Gao Li Zhang2015 PRC91-011603R Bao Gao Li Zhang2015 PRC92-034612 10^{-2} Liu_Shen_Li_Tu_Wang_Wang2016_EPJA52-35 Santhosh Safoora2016 PRC94-024623 Ghahramany Ansari2016 EPJA52-287 Hong Adamian Antonenko2016 EPJA52-305 Santhosh Safoora2017 PRC96-034610 Adamian Antonenko Lenske2018 NPA970-22

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A big issue in synthesizing SHE w/ Z>118



From Ch. E. Düllman (FUSHE2012)

Lectures 3 & 4

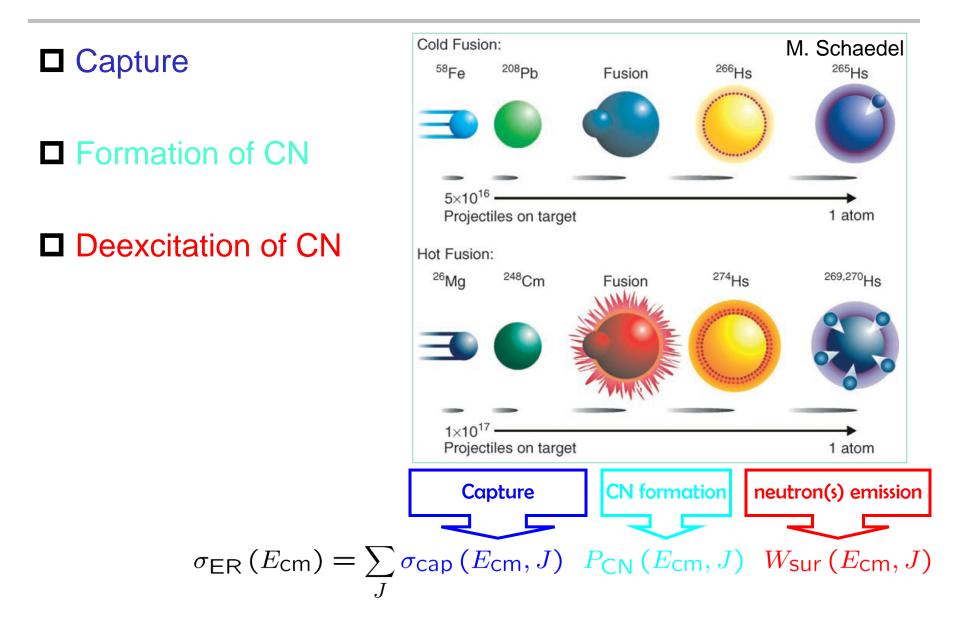
□ Challenges in synthesizing SHN

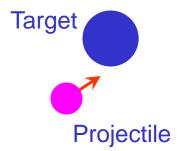
□ Synthesis mechanism of SHN

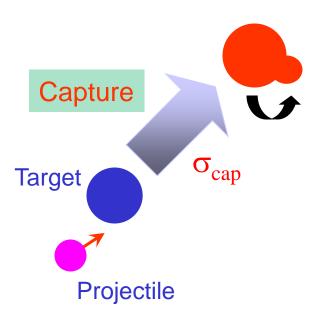
Large uncertainties in predicted Xsections

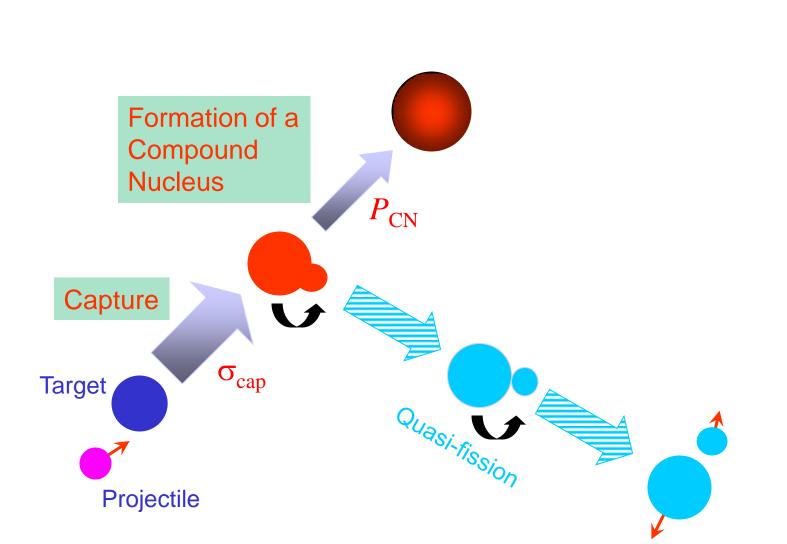
Heavy ion fusion reactions

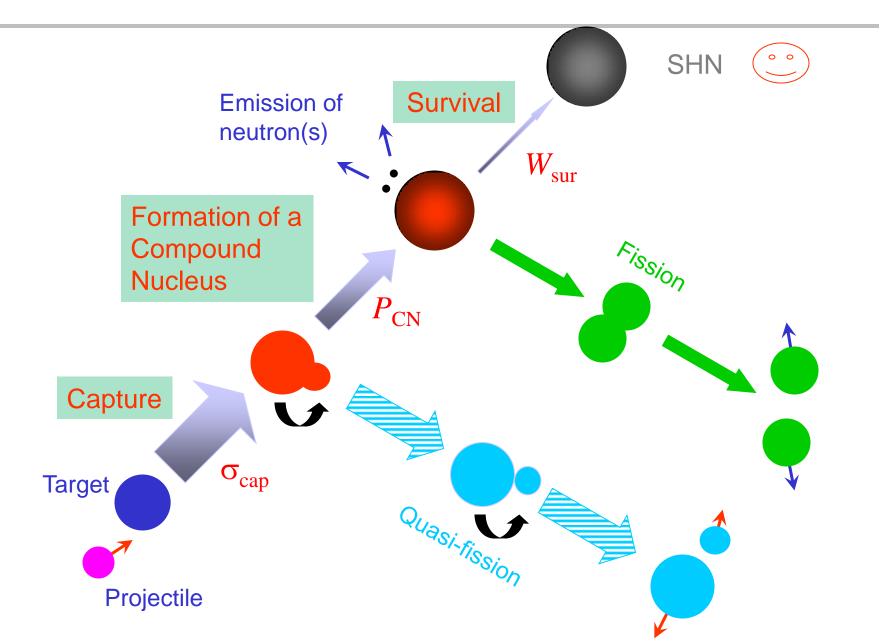
- Capture
- Fusion
- Survival against fission
- Multi-nucleon transfer reactions
- Reactions w/ radioactive ion beams











Lectures 3 & 4

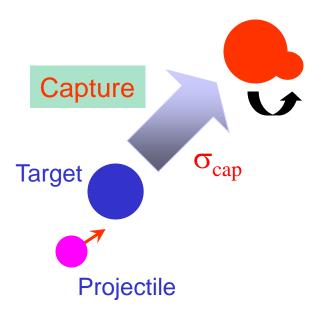
□ Challenges in synthesizing SHN

□ Synthesis mechanism of SHN

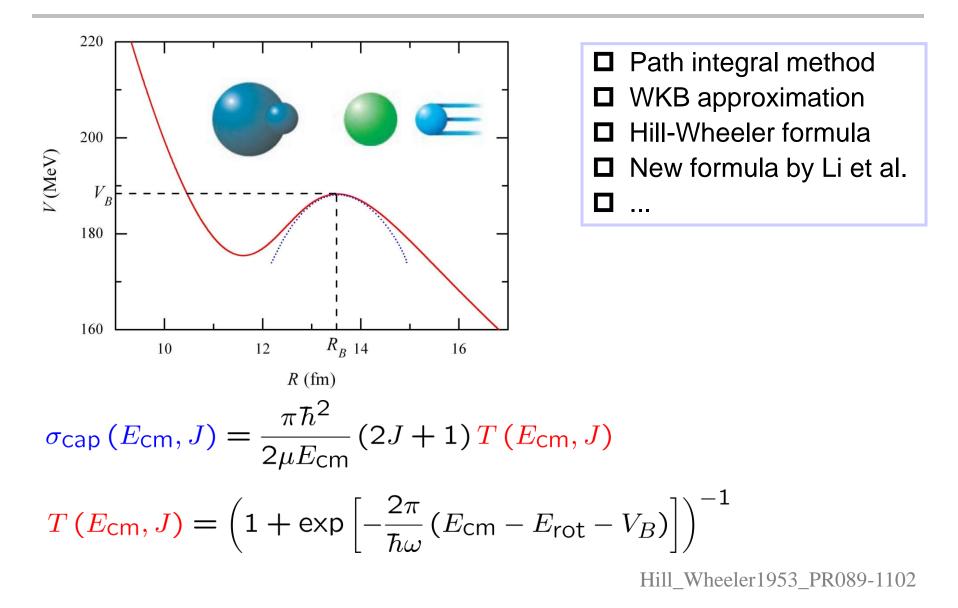
Large uncertainties in predicted Xsections

- Heavy ion fusion reactions
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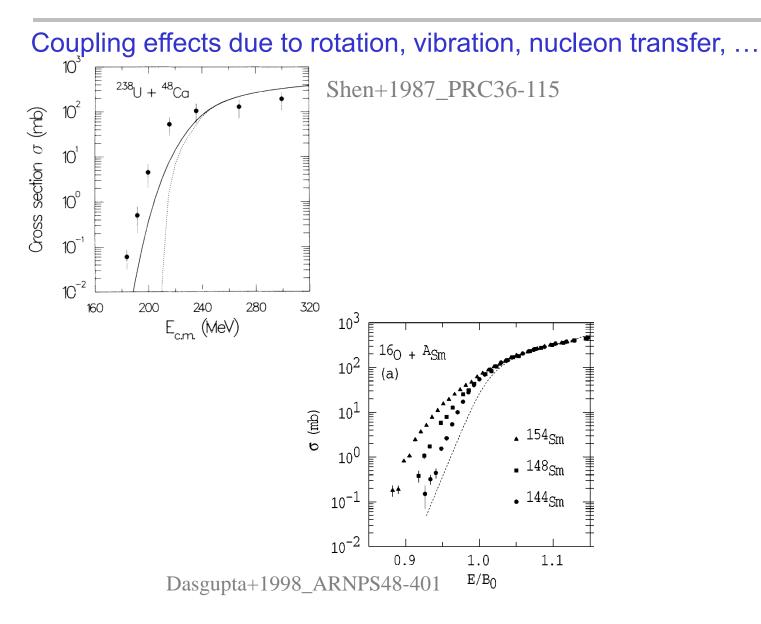




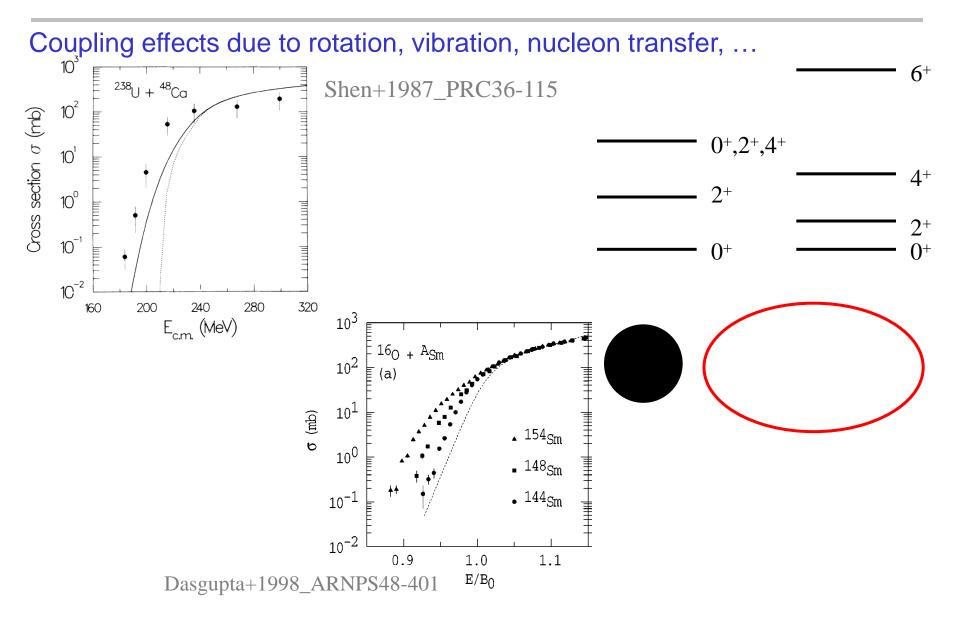
The capture process

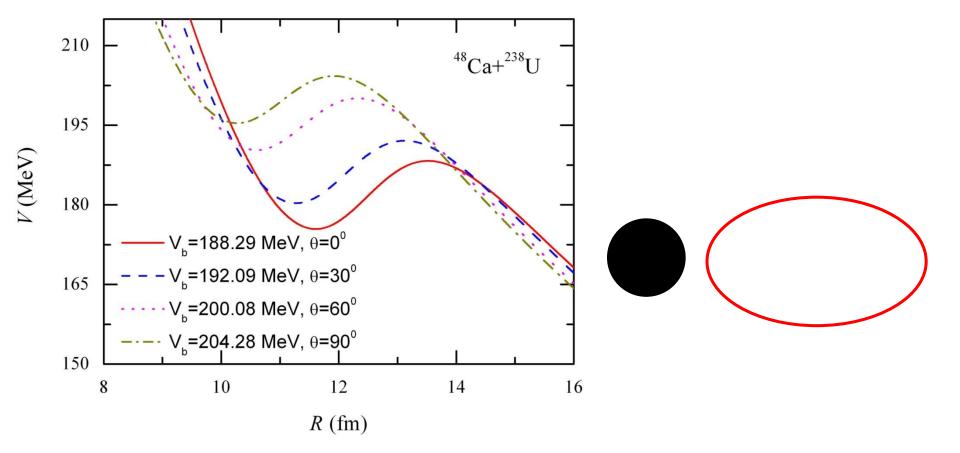


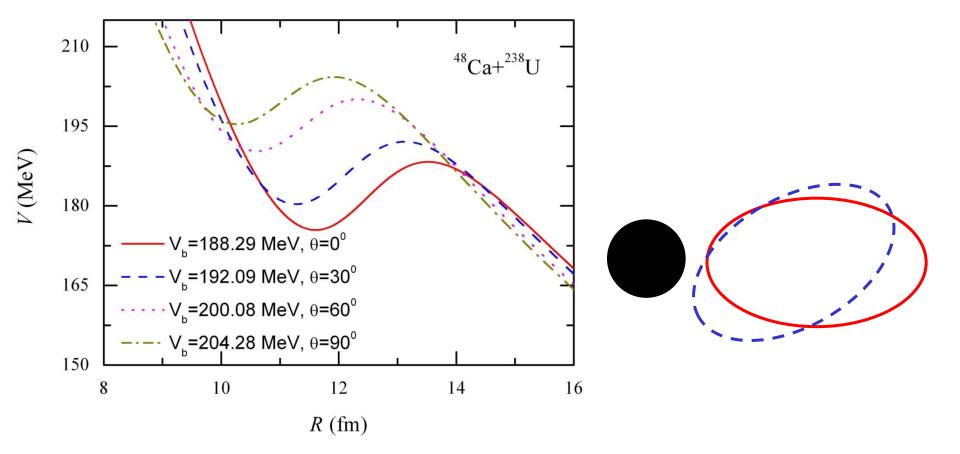
Channel coupling effects

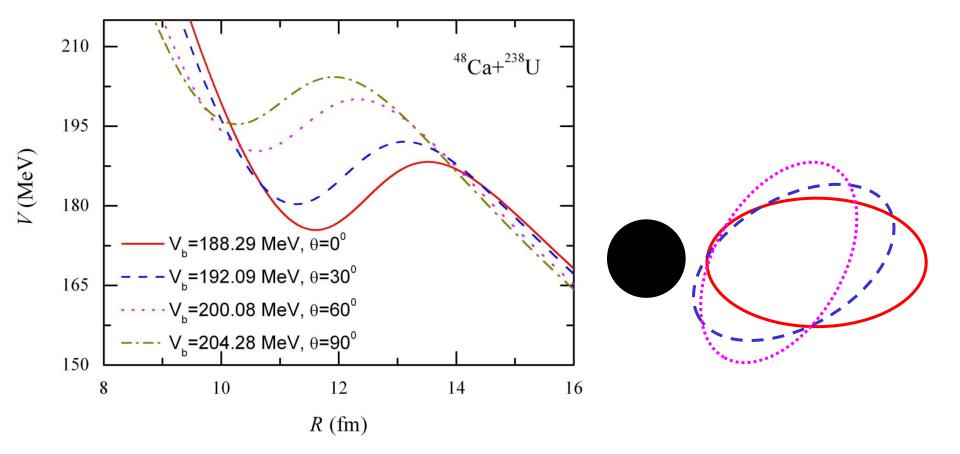


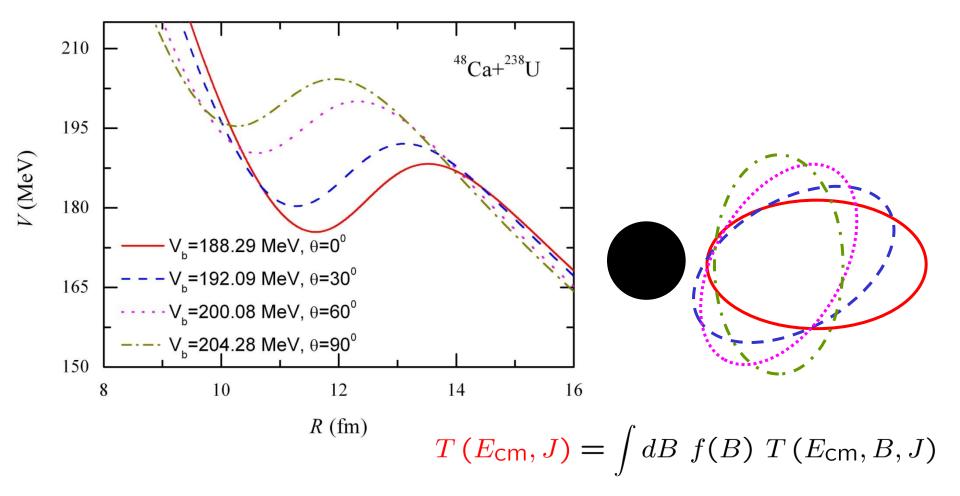
Channel coupling effects



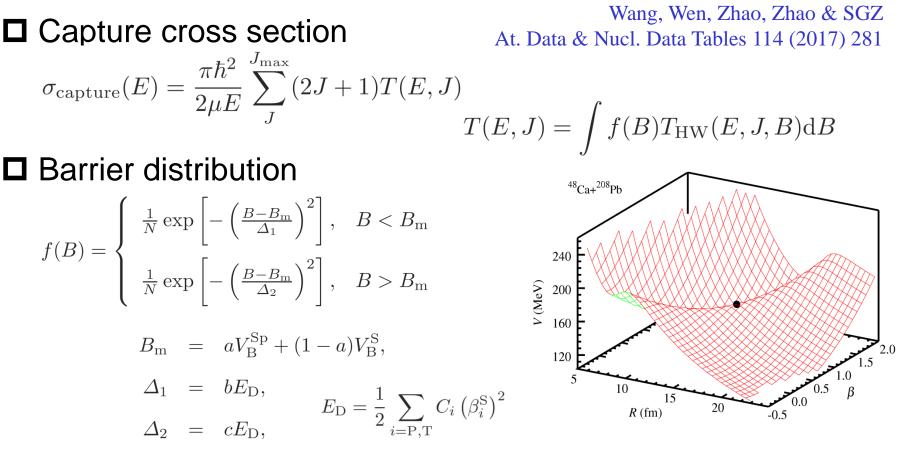








The empirical CC (ECC) model



For reactions w/ positive Q-values for 2n transfer

$$\Delta_1 \rightarrow gQ(2n) + \Delta_1$$

 $\Delta_2 \rightarrow gQ(2n) + \Delta_2$

Model parameters

Wang, Wen, Zhao, Zhao & SGZ At. Data & Nucl. Data Tables 114 (2017) 281

$$V_{\rm N}(R, \beta_{\rm P}^0, \beta_{\rm T}^0, \theta_{\rm P}, \theta_{\rm T}) = \frac{-V_0}{1 + \exp\left[(R - R_{\rm P} - R_{\rm T})/a\right]},$$

Nucleus-nucleus potential

with

$$R_{\rm P} = R_{0\rm P} [1 + (5/4\pi)^{1/2} \beta_{\rm P}^{0} P_2(\cos \theta_{\rm P})],$$

$$R_{\rm T} = R_{0\rm T} [1 + (5/4\pi)^{1/2} \beta_{\rm T}^{0} P_2(\cos \theta_{\rm T})],$$

$$R_{0i} = r_0 A_i^{1/3}, \quad i = \rm P, T,$$

$$V_0 = 80 \,\,{\rm MeV},$$

$$r_0 = 1.16 \,\,{\rm fm},$$

$$a = \left\{ 1.17 \left[1 + 0.53 \left(A_{\rm P}^{-1/3} + A_{\rm T}^{-1/3} \right) \right] \right\}^{-1} \,\,{\rm fm}.$$

Deformation parameters: Möller_Nix_Myers_Swiatecki_ADNDT59-185

Barrier distribution

Reactions w/ spherical T & P Reactions w/ deformed T or P

$$a = \begin{cases} 0.26, & Z_P Z_T < 1150, \\ 0.5, & Z_P Z_T \ge 1150, \\ b = 0.32, \\ c = 0.93. \end{cases} \qquad a = \begin{cases} 0.23, & Z_P Z_T < 1150, \\ 0.37, & Z_P Z_T \ge 1150, \\ b = 0.12, \\ c = 1.12. \end{cases} \qquad g = 0.32$$

Fusion reactions w/ well bound projectiles

□ 220 reactions with $182 \le Z_P Z_T \le 1640$

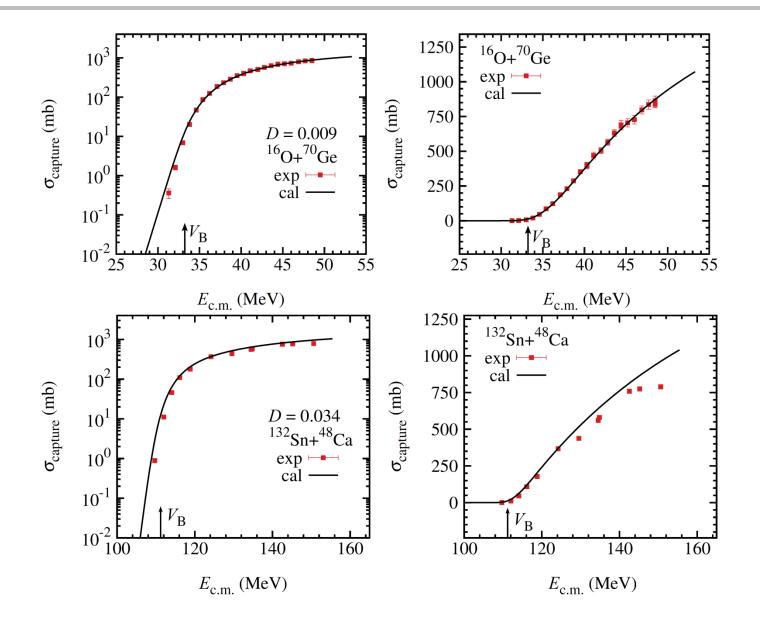
Table 2

Wang, Wen, Zhao, Zhao & SGZ At. Data & Nucl. Data Tables 114 (2017) 281

The experimental and calculated excitation functions for a total of 217 reaction systems. See page 56 for Explanation of Tables.

| Reaction | Detected particles | Data obtained | $E_{\rm lab}$ | $E_{\rm c.m.}$ | σ | $\delta\sigma$ | $\sigma^{ m cal}$ |
|-----------------------------------|----------------------|----------------|---------------|----------------|----------|----------------|-------------------|
| | | | (MeV) | (MeV) | (mb) | (mb) | (mb) |
| ¹² C+ ⁸⁹ Y | EvR | authors' graph | 30.682 | 27.037 | 0.033 | | 0.195 |
| | | [103] | 32.403 | 28.554 | 0.668 | | 1.697 |
| | | $E_{\rm lab}$ | 33.789 | 29.774 | 4.757 | | 9.135 |
| | | 1000 | 35.387 | 31.183 | 29.865 | | 47.223 |
| | | | 36.912 | 32.526 | 96.919 | | 132.929 |
| | | | 39.174 | 34.520 | 287.692 | | 302.095 |
| | | | 41.019 | 36.145 | 369.296 | | 434.894 |
| | | | 43.207 | 38.074 | 597.752 | | 575.375 |
| | | | 46.281 | 40.783 | 665.270 | | 744.850 |
| ¹² C+ ⁹² Zr | EvR | authors' table | 31.88 | 28.20 | 0.19 | 0.1 | 0.456 |
| | | [104] | 32.88 | 29.09 | 1.45 | 0.2 | 1.613 |
| | | $E_{\rm c.m.}$ | 33.89 | 29.98 | 2.91 | 0.2 | 5.554 |
| | | | 34.89 | 30.86 | 13.2 | 0.5 | 17.351 |
| | | | 35.89 | 31.75 | 38.6 | 1.1 | 45.991 |
| | | | 36.89 | 32.63 | 83.6 | 1.2 | 95.738 |
| | | | 37.89 | 33.52 | 136 | 3 | 163.147 |
| | | | 38.89 | 34.40 | 197 | 2 | 237.855 |
| | | | 39.90 | 35.30 | 253 | 3 | 315.353 |
| | | | 40.90 | 36.18 | 308 | 3 | 388.645 |
| | | | 41.91 | 37.07 | 366 | 3 | 459.032 |
| | | | 42.90 | 37.95 | 421 | 3 | 524.753 |
| | | | 43.89 | 38.83 | 476 | 4 | 586.786 |
| | | | 45.90 | 40.60 | 570 | 5 | 701.389 |
| | | | 47.91 | 42.38 | 664 | 5 | 804.554 |
| | | | 49.91 | 44.15 | 731 | 9 | 896.774 |
| $^{12}C + ^{144}Sm$ | EvR | authors' graph | 46.464 | 42.890 | 0.377 | | 0.815 |
| | | [105] | 46.968 | 43.355 | 0.975 | | 1.489 |
| | | $E_{\rm lab}$ | 47.407 | 43.760 | 1.458 | | 2.508 |

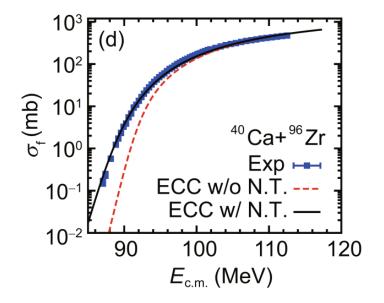
"Good" examples



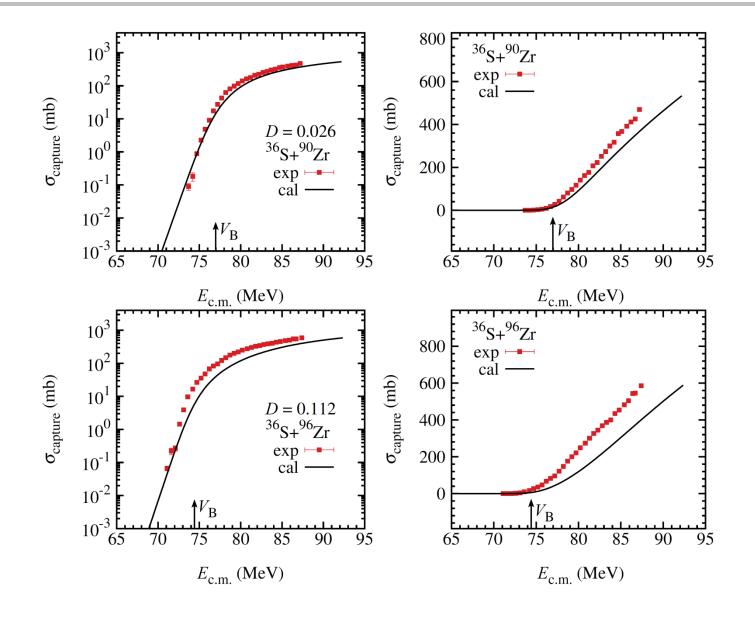
Wang, Zhao, Zhao & SGZ Sci. China-Phys. Mech. & Astron. 59 (2016) 642002

⁹⁶Zr

³²S(⁴⁰Ca)+

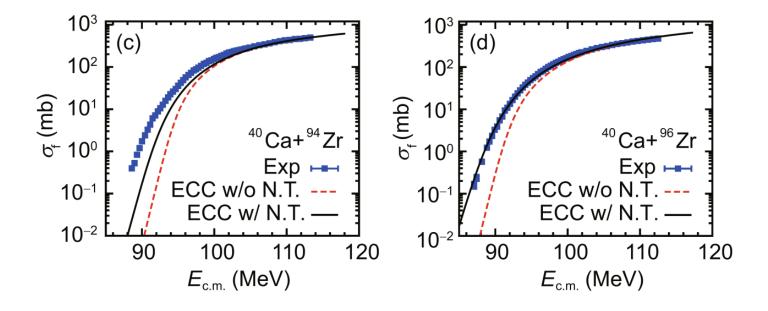


"Bad" examples



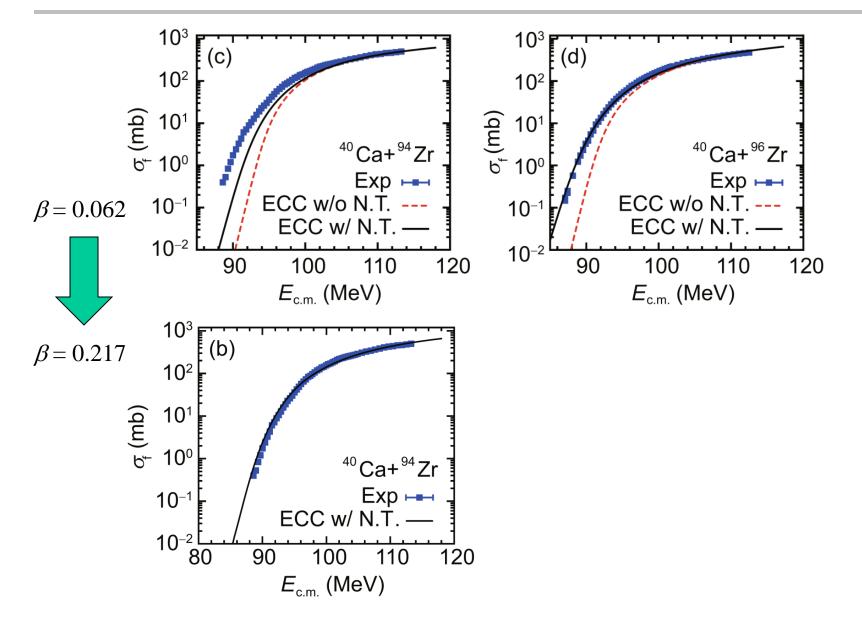


Wang, Zhao, Zhao & SGZ Sci. China-Phys. Mech. & Astron. 59 (2016) 642002

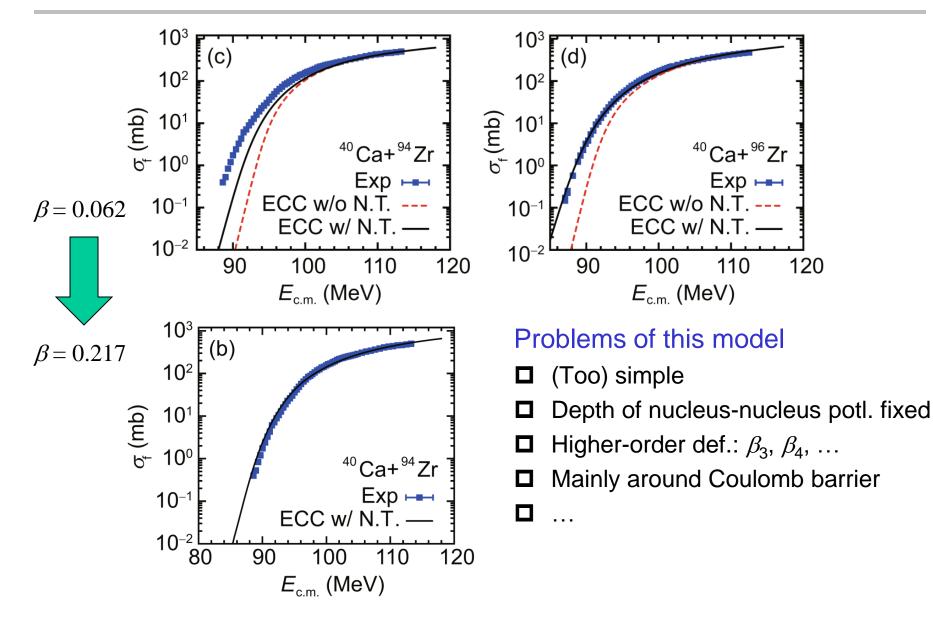




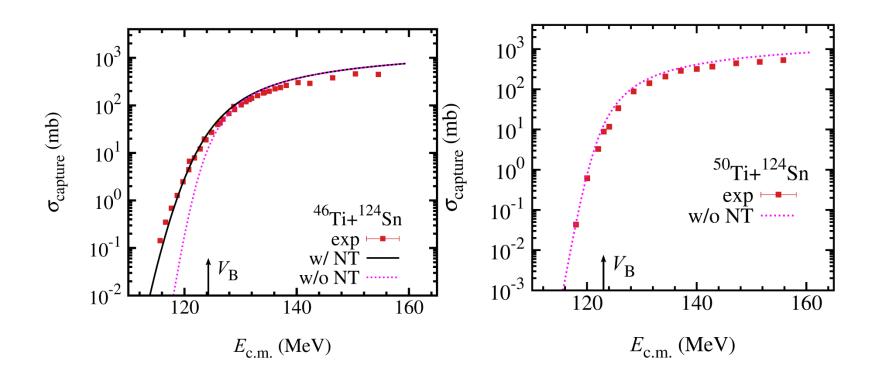
Wang, Zhao, Zhao & SGZ Sci. China-Phys. Mech. & Astron. 59 (2016) 642002







Predictions: ^{46,50}Ti+¹²⁴Sn



Expt: Liang+2016_PRC94-024616

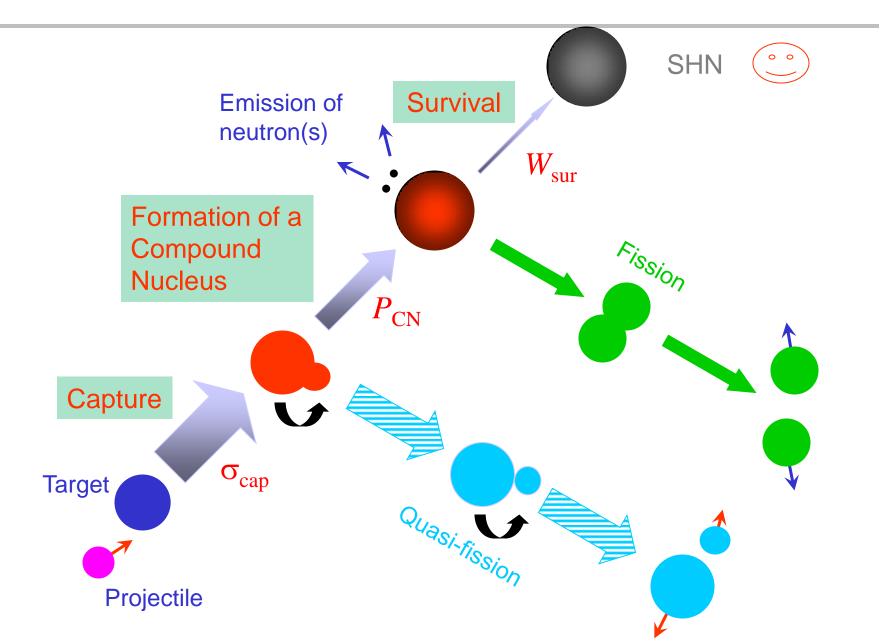
Lectures 3 & 4

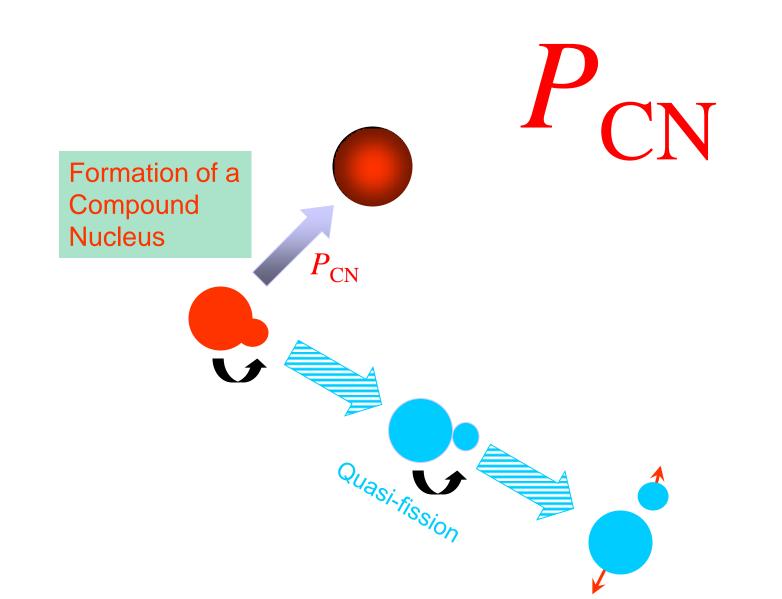
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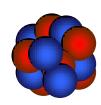
Large uncertainties in predicted Xsections

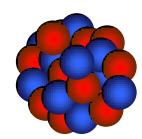
- Heavy ion fusion reactions
 - Capture
 - Fusion
 - Survival against fission
- Multi-nucleon transfer reactions
- Reactions w/ radioactive ion beams





Fusion of two many-body systems











Mac. DoFs

Relative distance

➤Mass & charge

Shape & orientation

➢Neck formation

Transfer of nucleons



Mac. DoFs

- Relative distance
- Mass & charge
- Shape & orientation
- Neck formation
- Transfer of nucleons

> ...

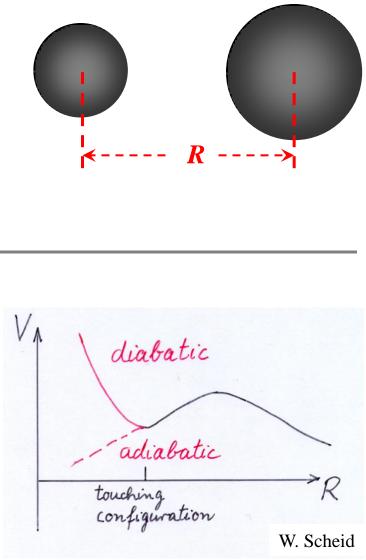
Langevin dynamics

$$-\frac{du(t)}{dt} = -\int_{-\infty}^{t} \gamma(t - t')u(t')dt' + \frac{1}{\mu}\delta F(t) - \frac{1}{\mu}\frac{dV(R)}{dR}$$
$$- u(t) = \frac{dR(t)}{dt} \quad \bullet R(t): \text{ Rel. distance}$$

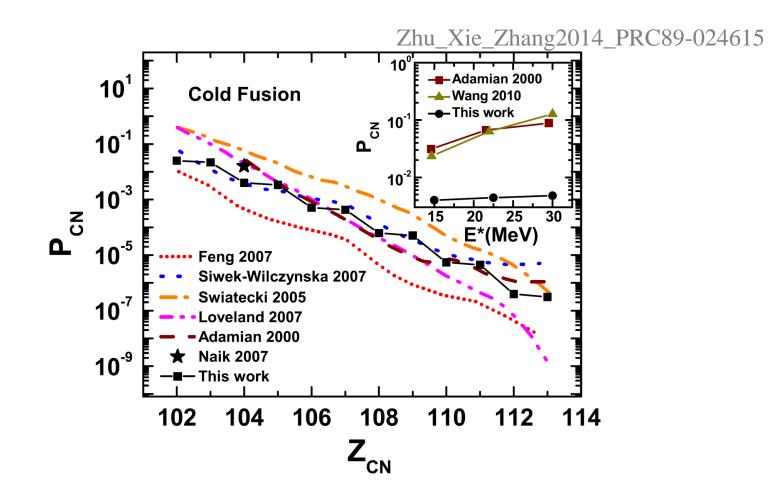
- R(t): Rel. distance
- *u*(*t*): Rel. velocity
- V(R): Interaction potl.
- d*F*(*t*): Random force
- g(*t*-*t*'): Friction force

Dinuclear system (DNS) model

- Projectile & target keep staying in the potl. Pocket and individuality
- Transfer of nucleons betw. Projectile & target may lead to fusion

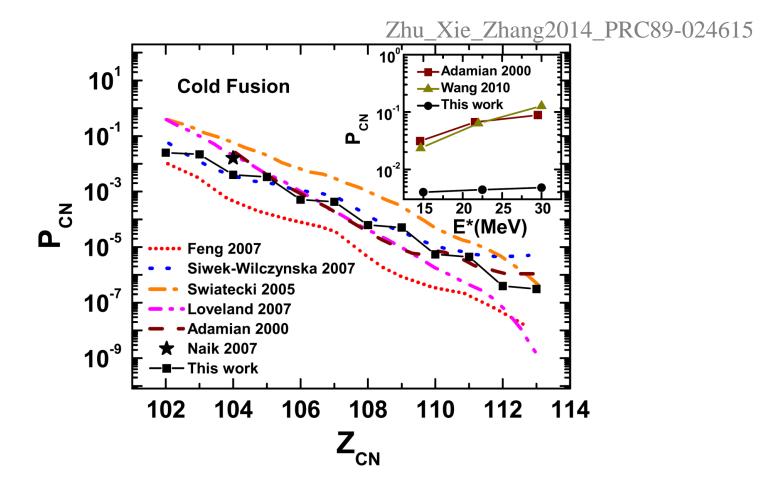


Large uncertainties in P_{CN} from various models



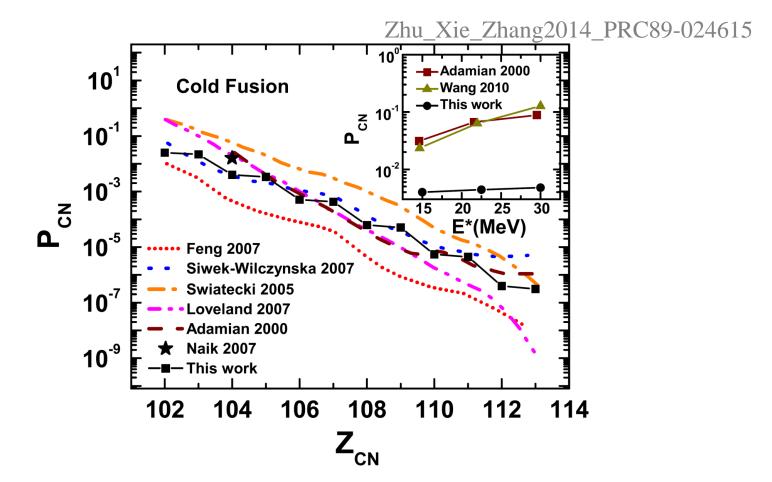
Large uncertainties in P_{CN} from various models

- **D** Experimental study? Systematics?
- Microscopic study of fusion mechanism



Large uncertainties in P_{CN} from various models

- **D** Experimental study? Systematics?
- Microscopic study of fusion mechanism





Mac. DoFs

Relative distance

➤Mass & charge

Shape & orientation

➢Neck formation

Transfer of nucleons



Mac. DoFs

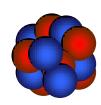
- Relative distance
- Mass & charge
- Shape & orientation
- Neck formation
- Transfer of nucleons

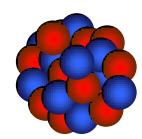
> ...





Fusion of two many-body systems



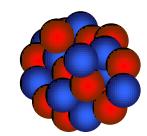


Fusion of two many-body systems

Time dependent Hartree-Fock theory

$$\mathcal{S} = \int_{t_1}^{t_2} dt \langle \Phi(t) | H - i\hbar \partial_t | \Phi(t) \rangle$$
$$H = \sum_i^A t_i + \sum_{i < j}^A v_{ij}$$
$$\Phi(r_1, r_2, \cdots, r_A, t) = \frac{1}{\sqrt{A!}} \det |\phi_\lambda(r_i, t)|$$





 $i\hbar\frac{\partial\phi_{\lambda}}{\partial t} = h\phi_{\lambda}$

Advantages

≻Microscopic

Negele 1982 Rev. Mod. Phys. 54-913 Guo...2007 Phys. Rev. C76-014601 Guo...2008 Phys. Rev. C77-041301(R)

Successful in structure & reaction

Simenel 2012 Euro. Phys. J. A 48-152

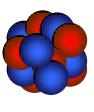
Disadvantages

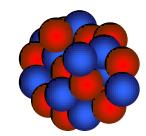
>Only one-body dissipation (collisions with "wall" of mean field)

Fusion of two many-body systems

Quantum Molecular Dynamics (QMD)

$$\phi_i(\boldsymbol{r}) = \frac{1}{\left(2\pi\sigma_r^2\right)^{3/4}} \exp\left[-\frac{(\boldsymbol{r}-\boldsymbol{r}_i)^2}{4\sigma_r^2} + \frac{i}{\hbar}\boldsymbol{r}\cdot\boldsymbol{p}_i\right]$$
$$\dot{\boldsymbol{r}}_i = \frac{\partial H}{\partial \boldsymbol{p}_i} , \quad \dot{\boldsymbol{p}}_i = -\frac{\partial H}{\partial \boldsymbol{r}_i}$$





Aichelin 1991 Phys. Rep. 202-233

Advantages

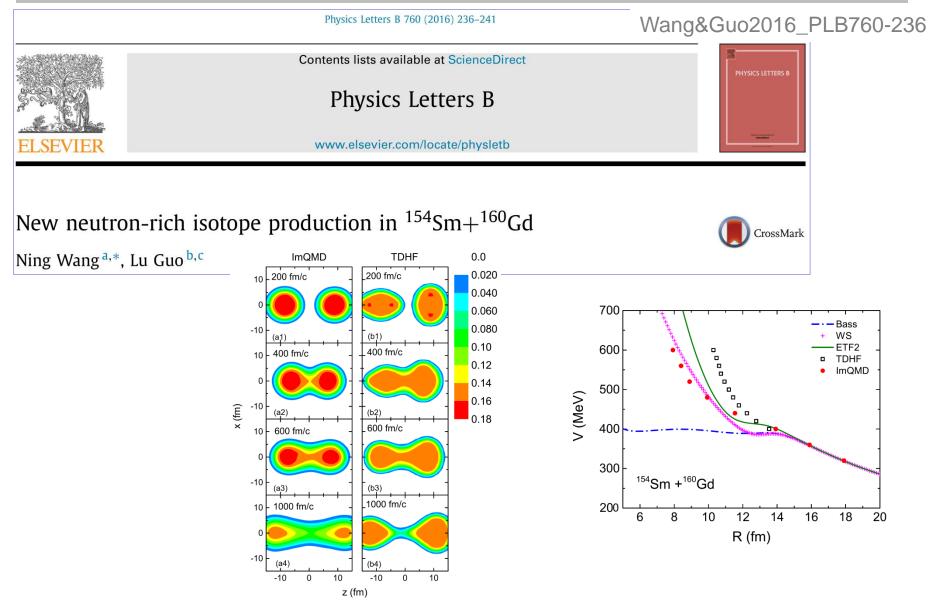
Wang_Li_Wu 2002 PRC65-064608 Wang...2004 PRC69-034608

≻Microscopic

➢Both mean field & collision terms included

- More attentions should be paid on
 - ➢Pauli exclusive principle
 - ≻Shell effects

ImQMD & TDHF: DIC & multi-nucleon transfer leading to n-rich isotopes



TDHF: quasifission dynamics

SCIENCE CHINA Physics, Mechanics & Astronomy

• Article •

September 2017 Vol. 60 No. 9: 092011 doi: 10.1007/s11433-017-9063-3

CrossMark

Angular momentum dependence of quasifission dynamics in the reaction ⁴⁸Ca+²⁴⁴Pu

Chong Yu, and Lu Guo*

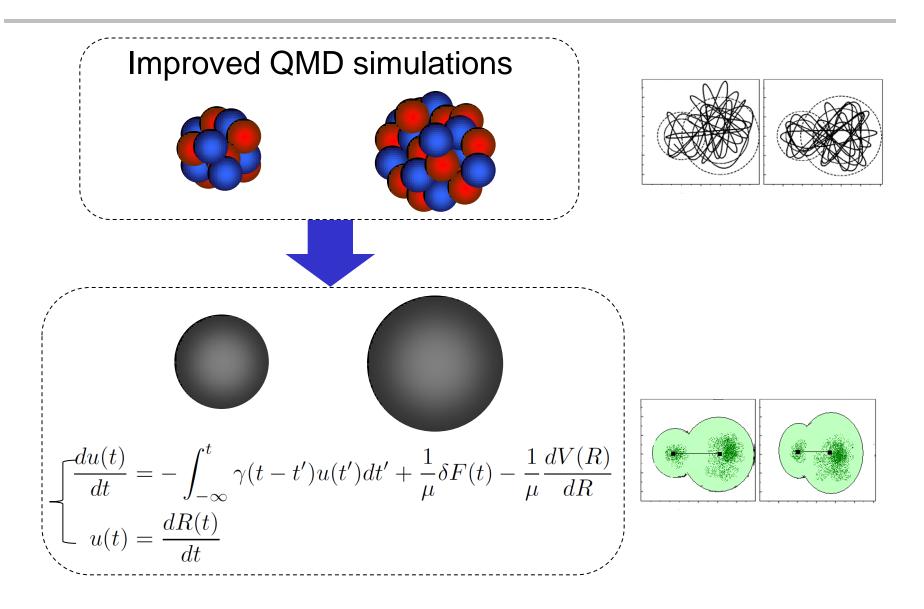
| • | (t=0.0 zs | *** t=0.93 zs | t=3.6 zs | 2 <i>t</i> =6.67 zs | to the Conta | ynamics is sensitive angular momentum act time of q.f. ases w/ the angular |
|----------|----------------------|-------------------------|--------------------|-------------------------------|-----------------|---|
| (t = | } =9.87 zs | <i>t</i> =10.67 zs | <i>t</i> =11.47 zs | t=12.0 zs | mome | Ŭ |

TDHF: fusion dynamics

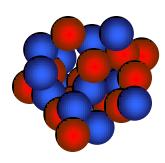


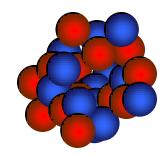
- Notable effects on fusion barriers & Xsections are observed by the inclusion of tensor force
- The effects are essentially attributed to the shift of low-lying vibration states of colliding partners & nucleon transfer in the asymmetric reactions

Fusion: Bridging microscopic & macroscopic description

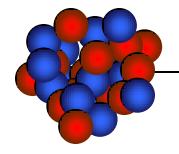


□ ⁹⁰Zr+⁹⁰Zr: Monte Carlo sampling; 10,000 events



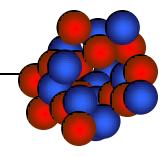


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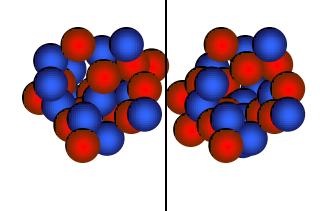


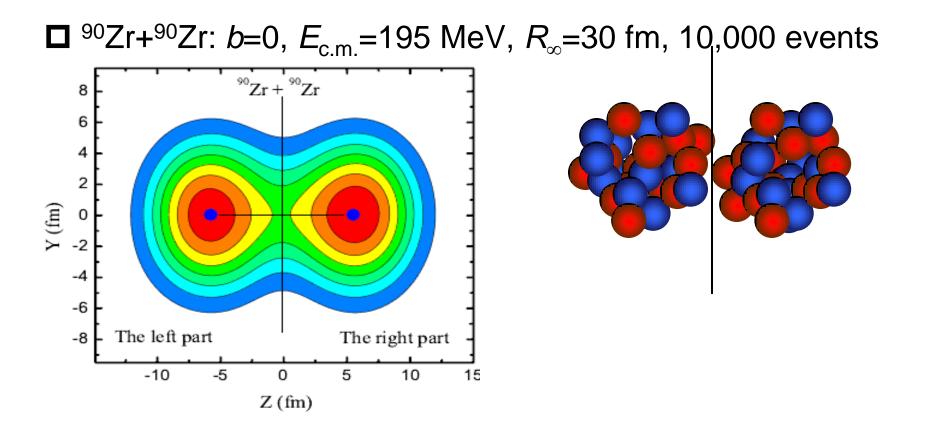
Central collision,
$$E_{\rm cm} = 195 \text{ MeV}$$

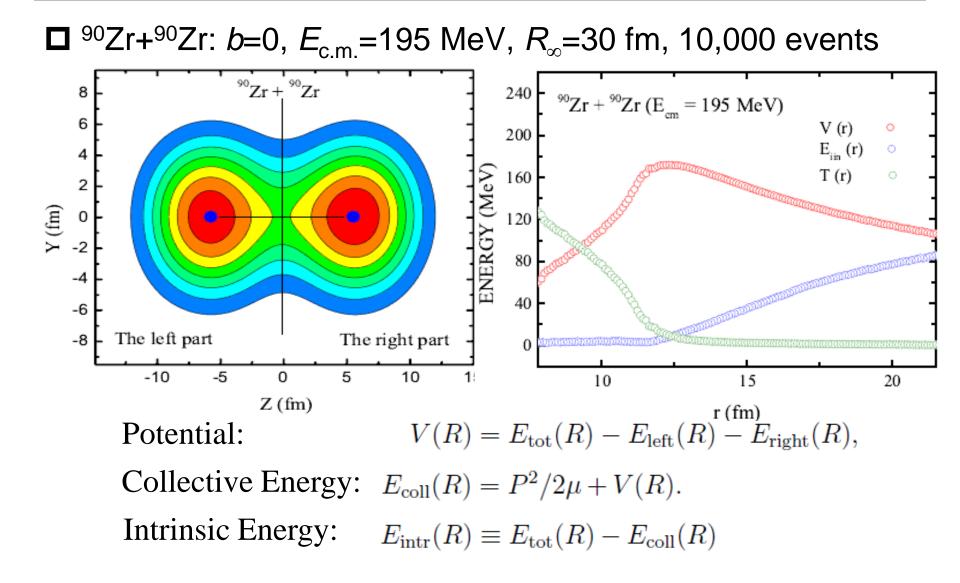
 $R_{\infty} = 30 \text{ fm}$



 \square ⁹⁰Zr+⁹⁰Zr: *b*=0, *E*_{c.m.}=195 MeV, *R*_∞=30 fm, 10,000 events

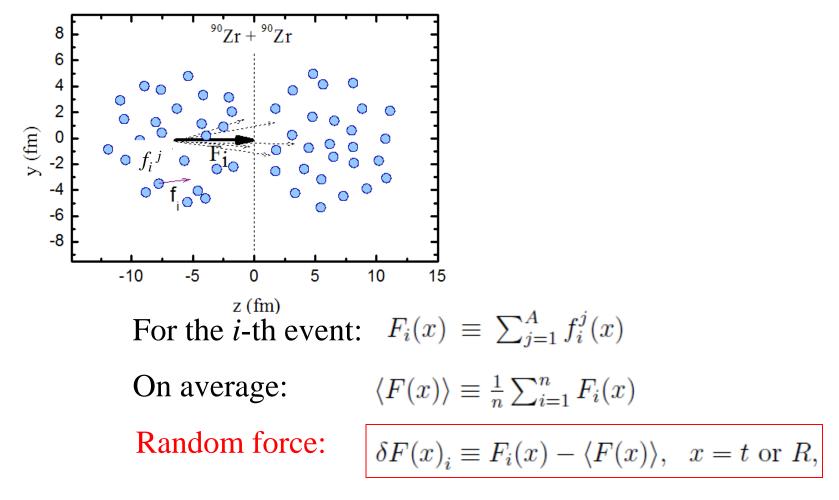






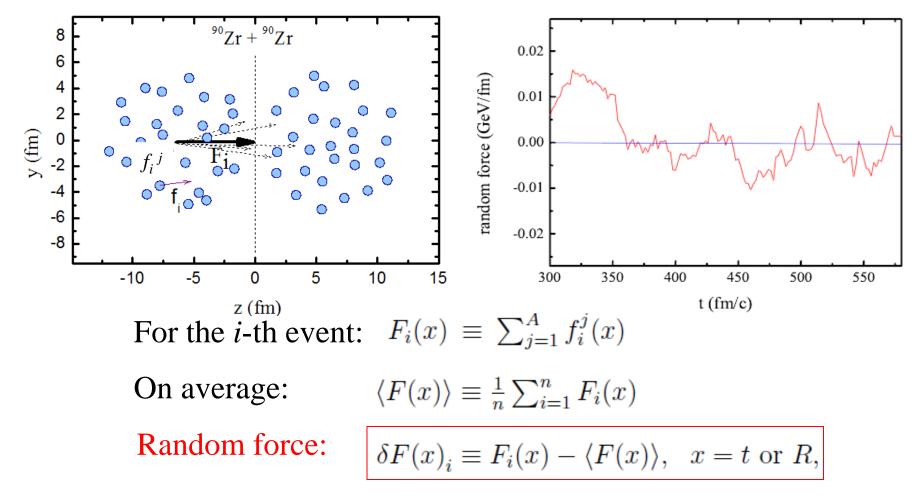
Macroscopic reduction of random force



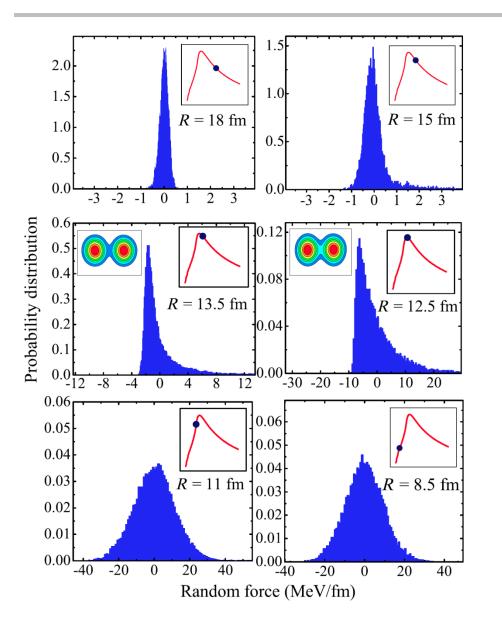


Macroscopic reduction of random force

Fluctuations from initialization & collisions



Distribution of random force



Approaching stage

Touching & fusion stage

Post-fusion stage

Wen_Sakata_Li_Wu_Zhang_SGZ 2013 Phys. Rev. Lett. 111, 012501

Fusion: Bridging microscopic & macroscopic description

- Macroscopic parameters, including the random force and the fricition coefficient, characterizing the Langevin type description of the nuclear fusion are extracted from the microscopic QMD dynamics around the Coulomb barrier
- The dissipation dynamics of the relative motion between two fusing nuclei is associated with non-Gaussian distributions of the random force
- A proper treatment of the non-Markovian (memory) effects in the Langevin dynamics is crutial for the dynamics of emergence in the nuclear dissipative fusion motion

Wen_Sakata_Li_Wu_Zhang_SGZ 2013 Phys. Rev. Lett. 111, 012501 Wen_Sakata_Li_Wu_Zhang_SGZ 2014 Phys. Rev. C90, 054613

Lectures 3 & 4

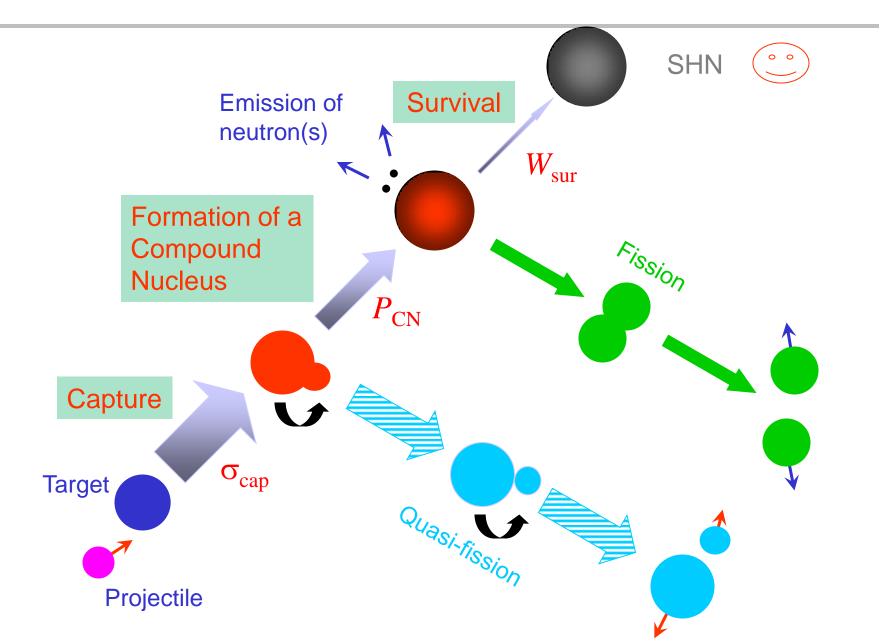
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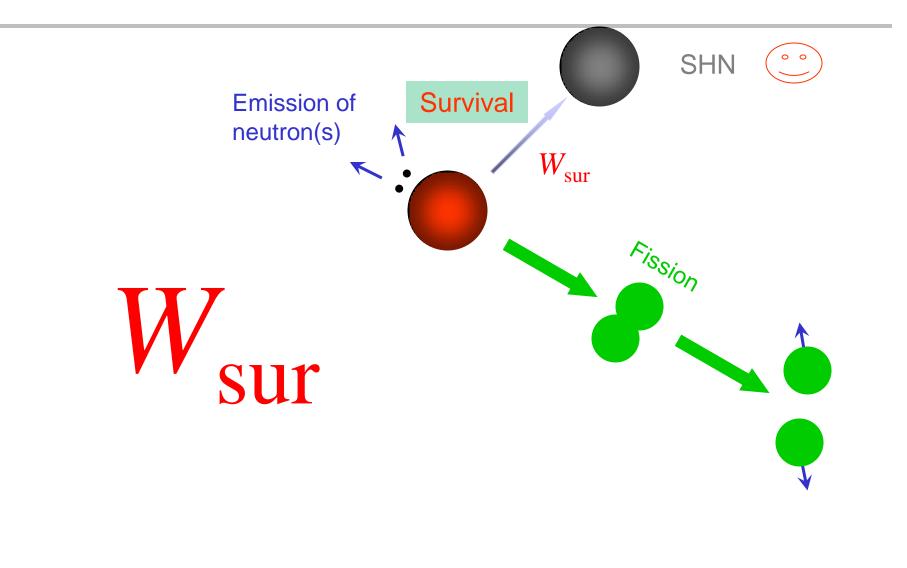
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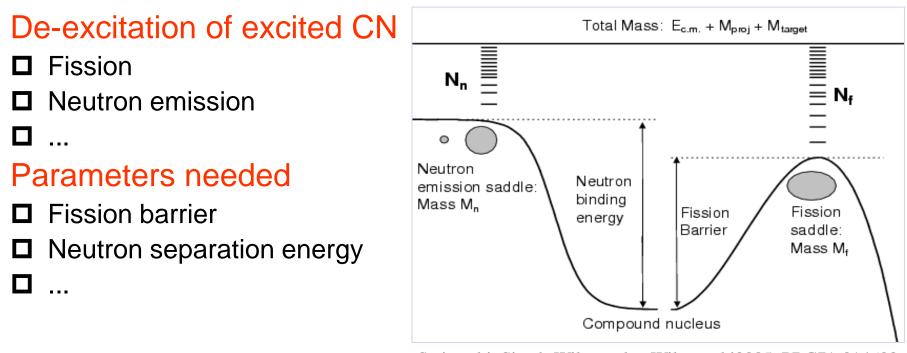
Three steps to a SHN via heavy-ion EvR reaction



Three steps to a SHN via heavy-ion EvR reaction



Fission of heavy & superheavy nuclei



Swiatecki_Siwek-Wilczynska_Wilczynski2005_PRC71-014602

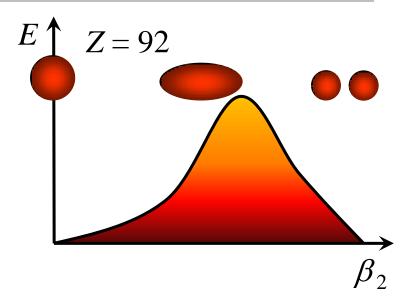
Theoretical approaches for calculating fission barriers:

- Macroscopic-microscopic approaches
- Self-consistent approaches

Multi-dim. constrained relativistic mean field models $(\beta_{20}, \beta_{22}, \beta_{30}, ...)$

Nuclear fission

Fission barrier is crucial for the description of fission

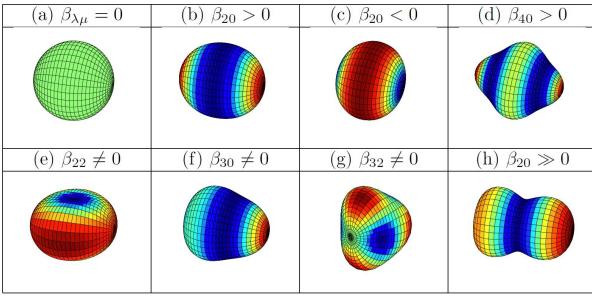


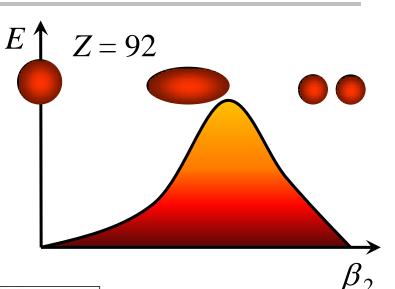
Courtesy of Bing-Nan Lu (吕炳楠)

Nuclear fission

- Fission barrier is crucial for the description of fission
- Various shapes may appear during fission

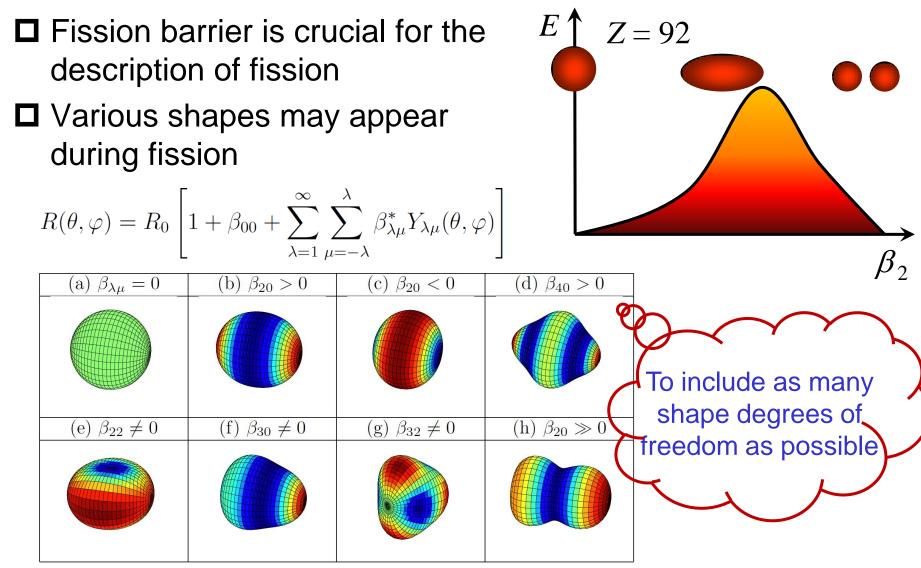
$$R(\theta,\varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta,\varphi) \right]$$





Courtesy of Bing-Nan Lu (吕炳楠)

Nuclear fission



Courtesy of Bing-Nan Lu (吕炳楠)

Covariant Density Functional Theory (CDFT)

$$\mathcal{L} = \bar{\psi}_{i} \left(i\partial - M \right) \psi_{i} + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) - g_{\sigma} \bar{\psi}_{i} \sigma \psi_{i} - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega^{\mu} - g_{\omega} \bar{\psi}_{i} \phi \psi_{i} - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_{\rho}^{2} \vec{\rho}_{\mu} \vec{\rho}^{\mu} - g_{\rho} \bar{\psi}_{i} \vec{\rho} \vec{\tau} \psi_{i}$$
 Serot_Walecka1986_ANP16-1
 - $\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_{i} \frac{1 - \tau_{3}}{2} \mathcal{A} \psi_{i},$ Reinhard1989 RP52-439
 - $\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_{i} \frac{1 - \tau_{3}}{2} \mathcal{A} \psi_{i},$ Ring1996_PPNP37-193
 Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101
 Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470
 ($\boldsymbol{\alpha} \cdot \mathbf{p} + \beta (M + S(\mathbf{r})) + V(\mathbf{r})) \psi_{i} = \epsilon_{i} \psi_{i}$ Liang_Meng_SGZ2015_PR570-1
 $\left(-\nabla^{2} + m_{\sigma}^{2} \right) \sigma = -g_{\sigma} \rho_{S} - g_{2} \sigma^{2} - g_{3} \sigma^{3}$ ($-\nabla^{2} + m_{\omega}^{2} \right) \omega = g_{\omega} \rho_{V} - c_{3} \omega^{3}$

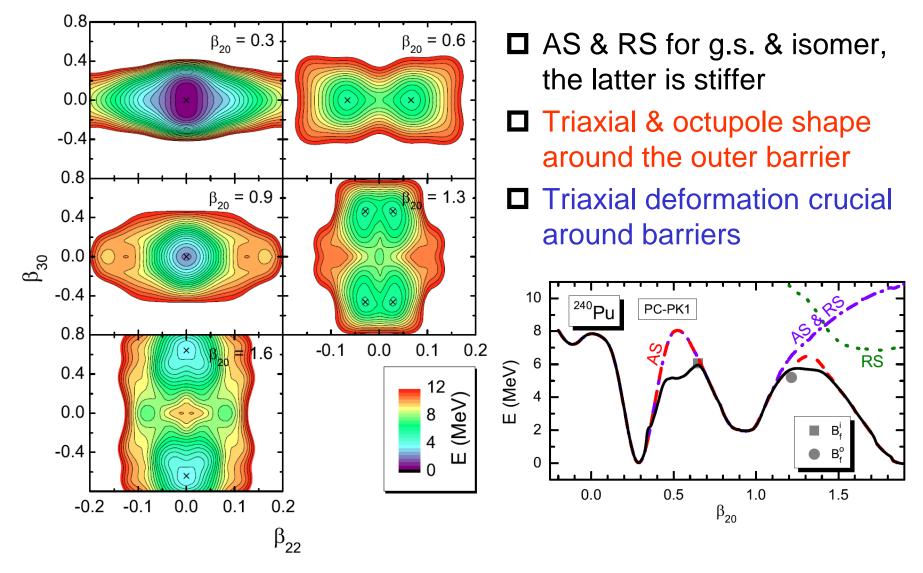
 $-\nabla^2 A = e\rho_C$

MDC-CDFT (β_{20} , β_{22} , β_{30} , β_{32} , β_{40} , ...)

| ph channel | Non-linear | Density-dependent | |
|-------------------|-----------------|-------------------|--|
| Meson exchange | NL3, NL3*, PK1, | DD-ME1, DD-ME2, | |
| Point Coupling | PC-F1, PC-PK1, | DD-PC1, | |
| | MDC-RMF | MDC-RHB | |
| pp channel | BCS | Bogoliubov | |
| Constant gap | \checkmark | | |
| Constant strength | \checkmark | | |
| Delta force | \checkmark | \checkmark | |
| Separable force | \checkmark | \checkmark | |

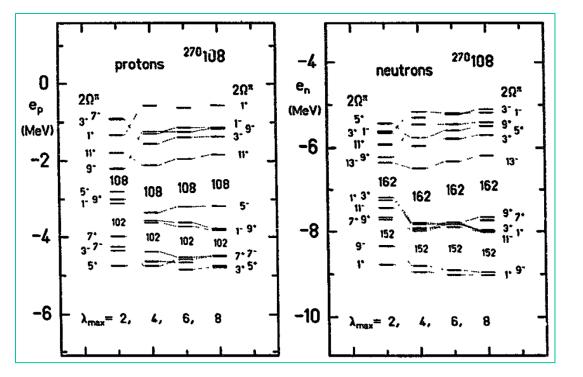
Lu_Zhao_SGZ 2011_PRC84-014328 Zhao_Lu_Zhao_SGZ 2012_PRC86-057304 Lu_Zhao_SGZ 2012_PRC85-011301R Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

²⁴⁰Pu: 3-dim. PES ($\beta_{20}, \beta_{22}, \beta_{30}$)



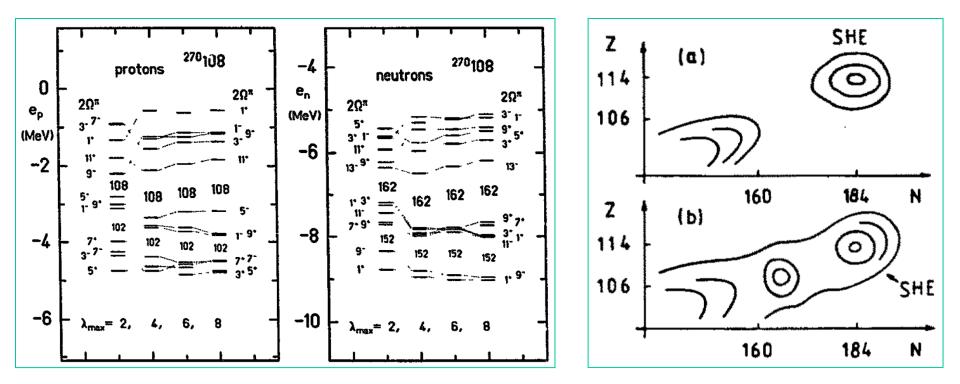
Lu_Zhao_SGZ 2012_PRC85-011301R

²⁷⁰Hs: A doubly magic deformed SHN



Patyk_Sobiczewski1991_NPA533-132

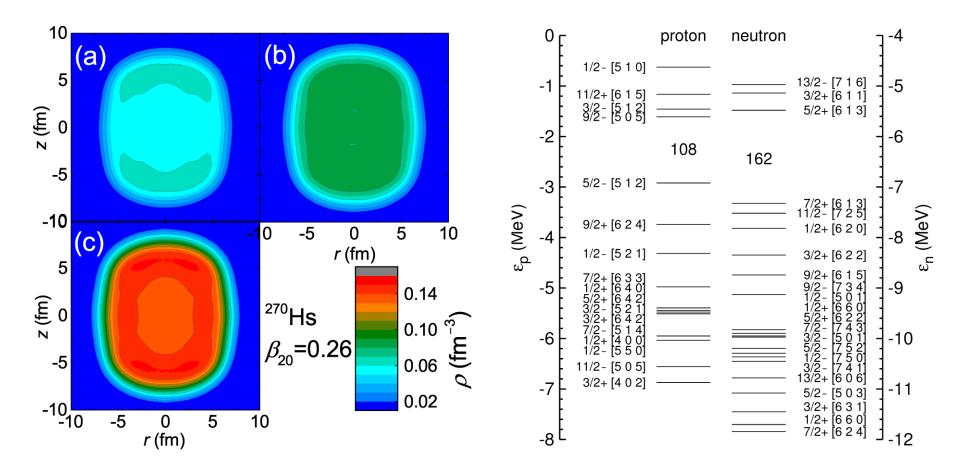
²⁷⁰Hs: A doubly magic deformed SHN



Patyk_Sobiczewski1991_NPA533-132

Patyk_Skalski_Sobiczewski_Cwiok 1989_NPA502-591c

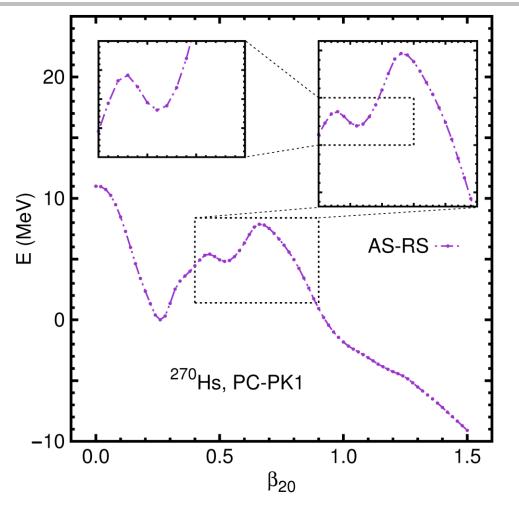
²⁷⁰Hs: ground state from MDC-RMF calc.



Xu Meng (孟旭), PhD thesis

Meng_Lu_Zhou2020 Sci. China-Phys. Mech. Astron. 63, 212011

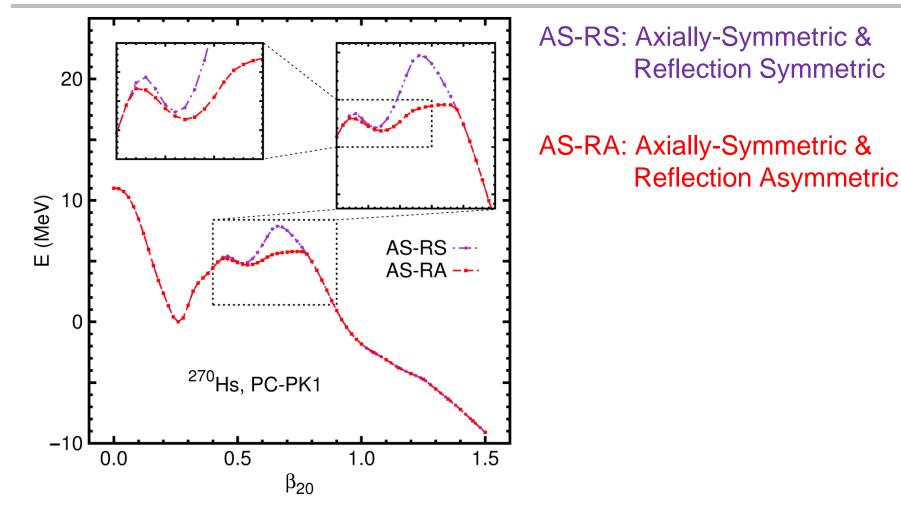
²⁷⁰Hs: 1D PEC from MDC-RMF calc.



AS-RS: Axially-Symmetric & Reflection Symmetric

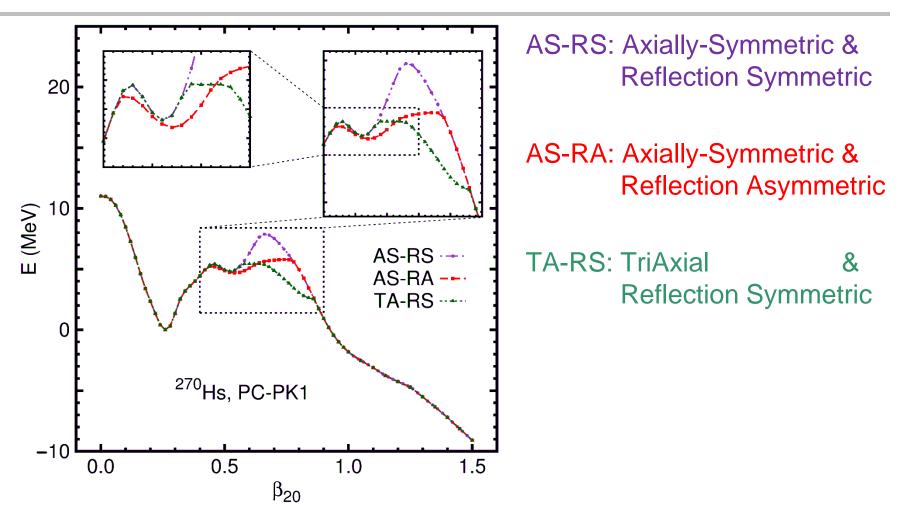
Courtesy of Xu Meng (孟旭)

²⁷⁰Hs: 1D PEC from MDC-RMF calc.



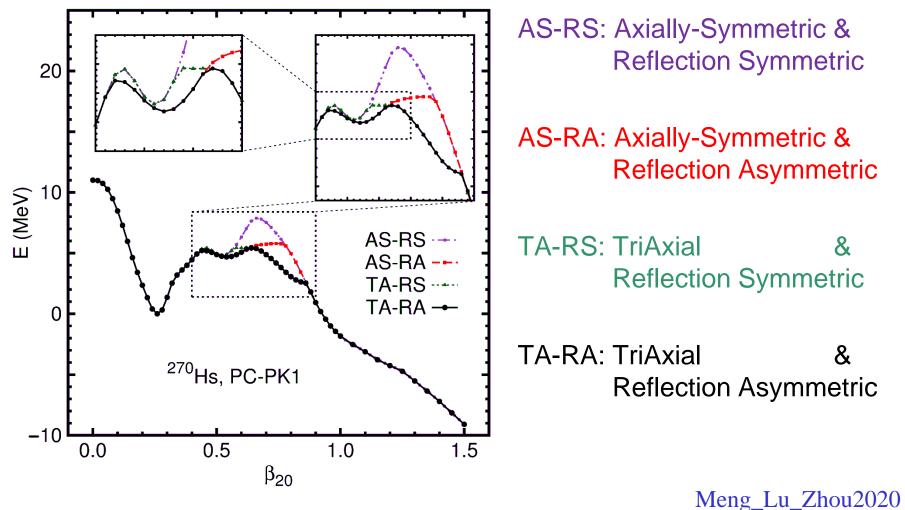
Courtesy of Xu Meng (孟旭)

²⁷⁰Hs: 1D PEC from MDC-RMF calc.



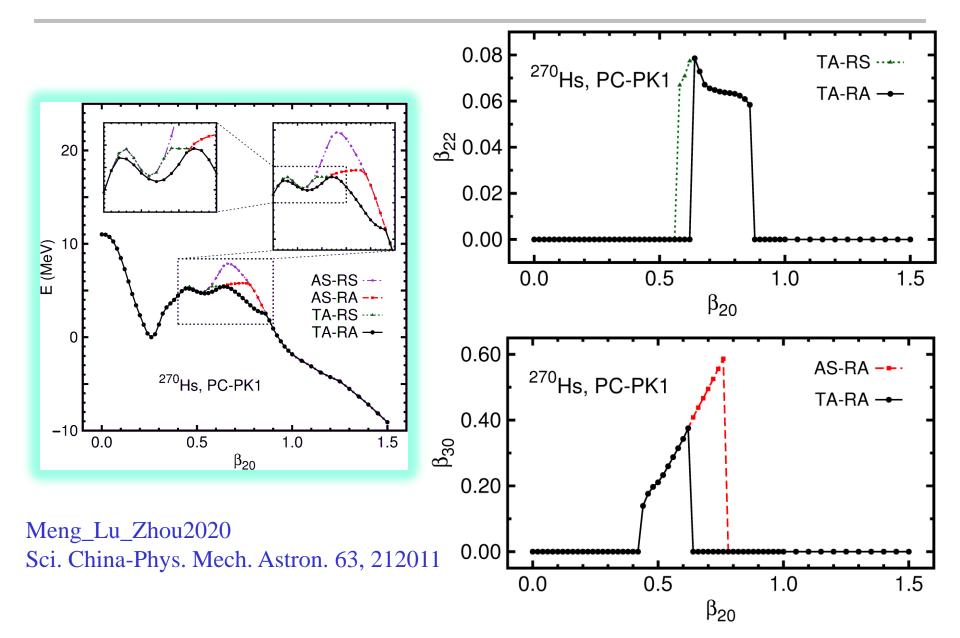
Courtesy of Xu Meng (孟旭)

²⁷⁰Hs: 1D PEC from MDC-RMF calc.

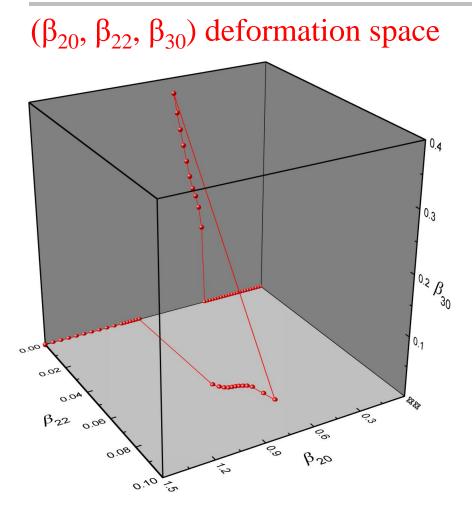


Sci. China-Phys. Mech. Astron. 63, 212011

Discontinuities in 1D PECs



²⁷⁰Hs: 1D TA-RA "PEC" viewed in 3D def. space



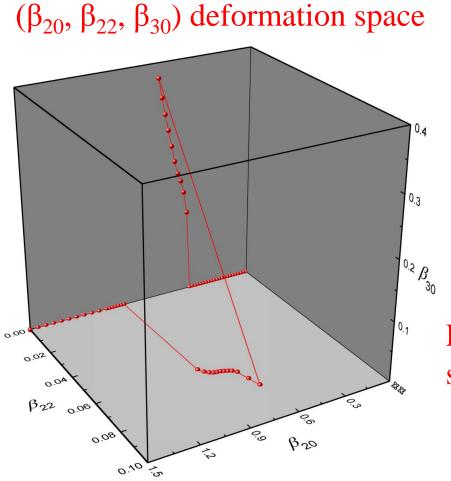
Courtesy of Xu Meng (孟旭)

TA-RA "PEC" consists of 4 segments:

- $β_{20} = (0 , 0.42), β_{22} = 0, β_{30} = 0$

$$\succ$$
 β₂₀ = (0.88, 1.50), β₂₂ = 0, β₃₀ = 0

²⁷⁰Hs: 1D TA-RA "PEC" viewed in 3D def. space



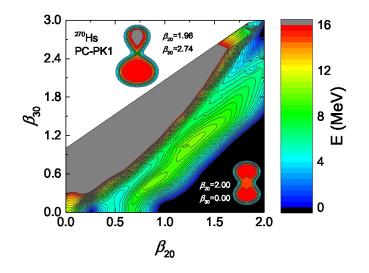
TA-RA "PEC" consists of 4 segments:

$$▶ β_{20} = (0.64, 0.86), β_{22} ≠ 0, β_{30} = 0$$

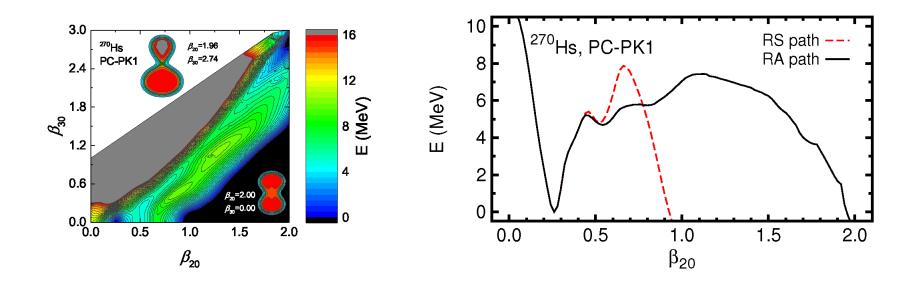
$$\succ \ \beta_{20} = (0.88, 1.50), \, \beta_{22} = 0, \, \beta_{30} = 0$$

How can a nucleus change its shape so suddenly ?

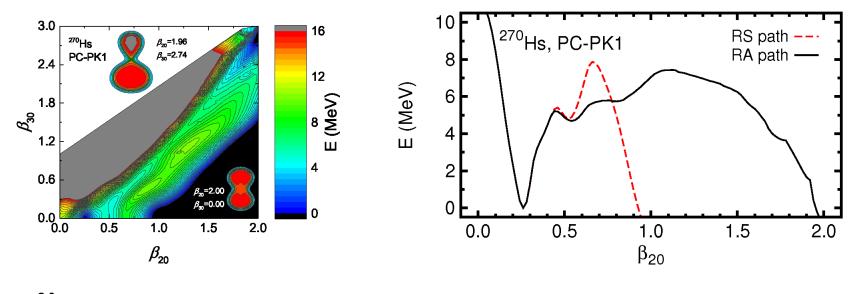
Courtesy of Xu Meng (孟旭)

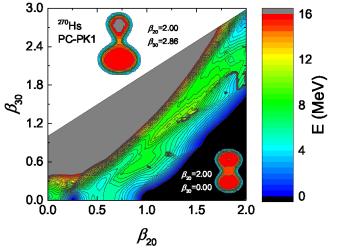


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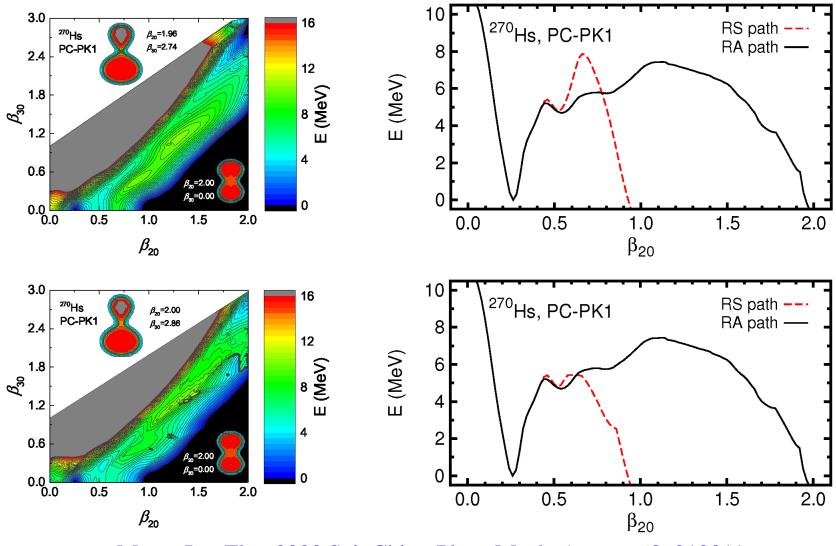


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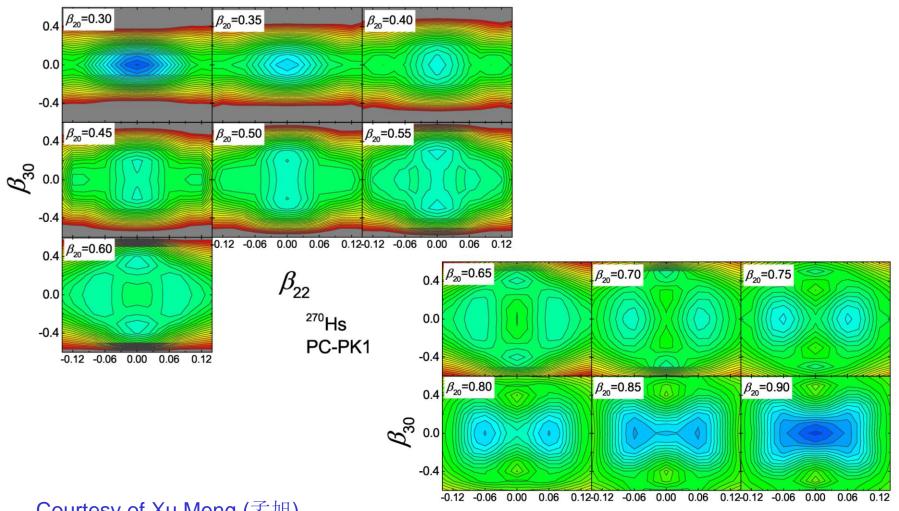


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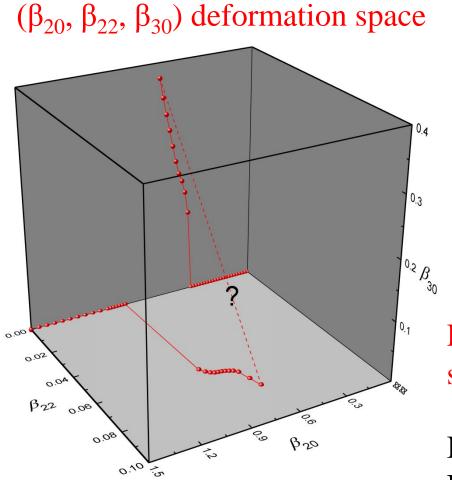
Meng_Lu_Zhou2020 Sci. China-Phys. Mech. Astron. 63, 212011

²⁷⁰Hs: 3D PES



Courtesy of Xu Meng (孟旭)

²⁷⁰Hs: 1D TA-RA "PEC" viewed in 3D def. space



Courtesy of Xu Meng (孟旭)

TA-RA "PEC" consists of 4 segments:

- \succ β₂₀ = (0 , 0.42), β₂₂ = 0, β₃₀ = 0
- β_{20} = (0.44, 0.62), β_{22} = 0, β_{30} ≠ 0

$$▶ β_{20} = (0.64, 0.86), β_{22} ≠ 0, β_{30} = 0$$

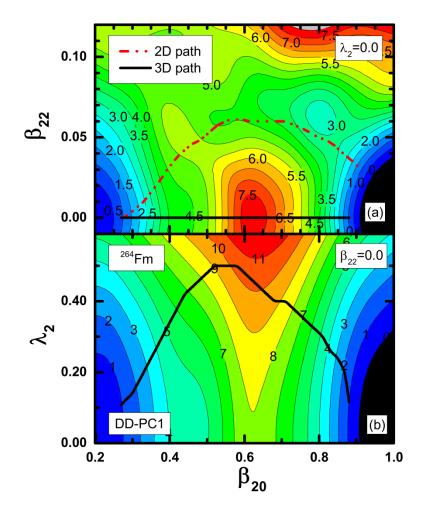
$$\succ$$
 β₂₀ = (0.88, 1.50), β₂₂ = 0, β₃₀ = 0

How can a nucleus change its shape so suddenly ?

It may do it through when shape DoFs are considered explicitly !

MDC-RMF study of spontaneous fission: Coupling between shape and pairing

WKB w/ action in coll. spaces based on PES from MDC-RMF



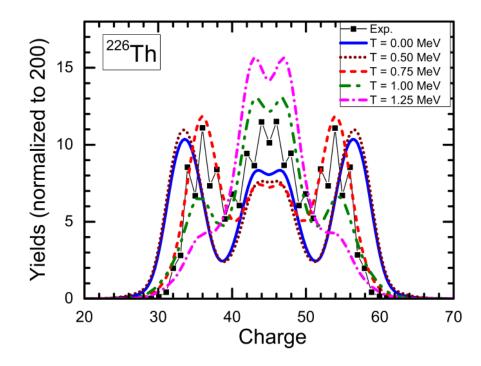
Pairing favors axially symmetric shapes along the dynamic path of the fissioning system, amplifies pairing as the path traverses the fission barriers, significantly reduces the action integral, and shortens the corresponding SF half-life.

Zhao_Lu_Niksic_Vretenar_SGZ 2016_PRC93-044315

Micro. self-consistent description of induced fission dyn.: finite temperature (FT) effects

TDGCM + GOA based on PES from FT MDC-RMF

- The model can qualitatively reproduce the empirical triple-humped structure of the fission charge and mass distributions at T = 0
- But the precise expt. position of the asymmetric peaks & the symmetricfission yield can only be accurately reproduced when FT effects included



Zhao_Niksic_Vretenar_SGZ 2018_arXiv1809.06114

Lectures 3 & 4

□ Challenges in synthesizing SHN

□ Synthesis mechanism of SHN

Large uncertainties in predicted Xsections

Heavy ion fusion reactions

Capture

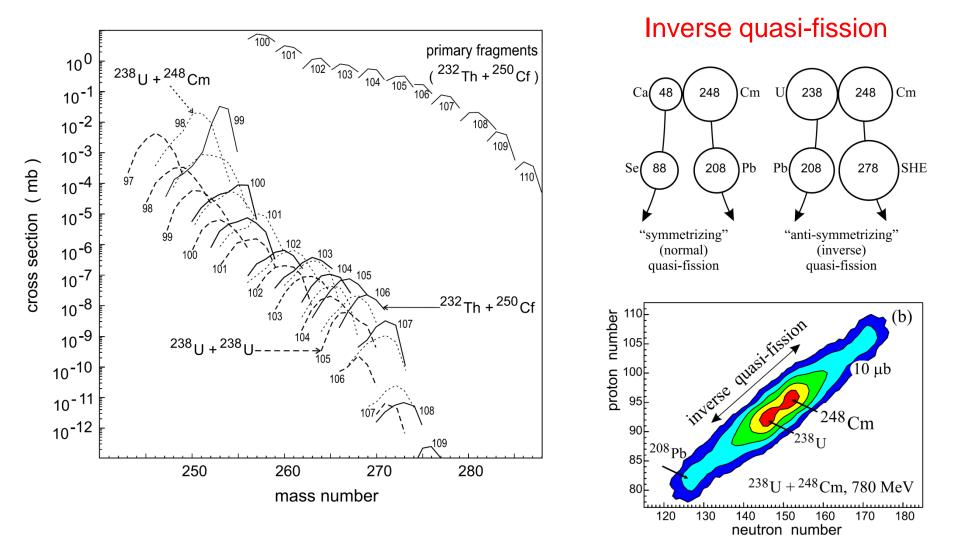
Fusion

Survival against fission

Multi-nucleon transfer reactions

Reactions w/ radioactive ion beams

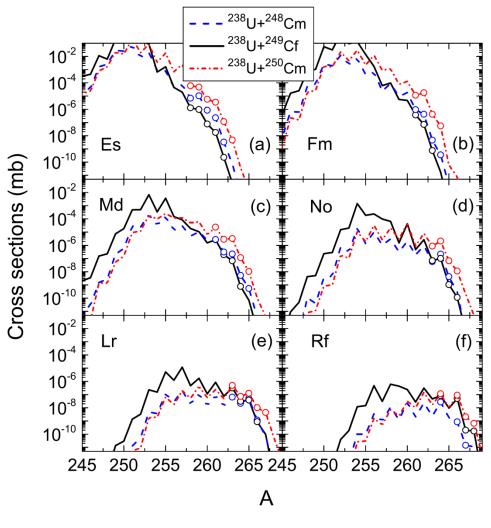
Multi-nucleon transfer reactions: Langevin dynamics



Zagrebaev+2006_PRC73-031602R

Zagrebaev&Greiner2015_NPA944-257

Multi-nucleon transfer reactions: DNS model



Zhu+2016_PRC94-054606

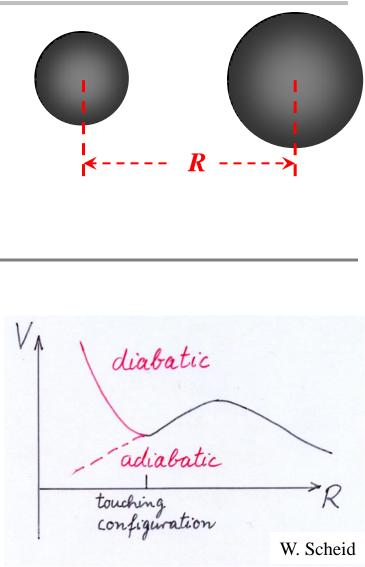
Fusion of two charged liquid drops

Langevin dynamics $\int \frac{du(t)}{dt} = -\int_{-\infty}^{t} \gamma(t - t')u(t')dt' + \frac{1}{\mu}\delta F(t) - \frac{1}{\mu}\frac{dV(R)}{dR}$ $u(t) = \frac{dR(t)}{dt} \quad \bullet R(t): \text{ Rel. distance}$ • R(t): Rel. distance

- *u*(*t*): Rel. velocity
- V(R): Interaction potl.
- d*F*(*t*): Random force
- g(*t*-*t*'): Friction force

Dinuclear system (DNS) model

- Projectile & target keep staying in the potl. Pocket and individuality
- Transfer of nucleons betw. Projectile & target may lead to fusion



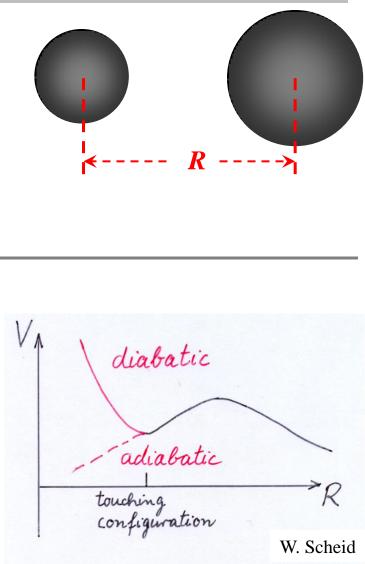
Nucleon(s) transfer betw. two charged liquid drops

Langevin dynamics $\int \frac{du(t)}{dt} = -\int_{-\infty}^{t} \gamma(t - t')u(t')dt' + \frac{1}{\mu}\delta F(t) - \frac{1}{\mu}\frac{dV(R)}{dR}$ $u(t) = \frac{dR(t)}{dt} \quad \bullet R(t): \text{ Rel. distance}$ • R(t): Rel. distance

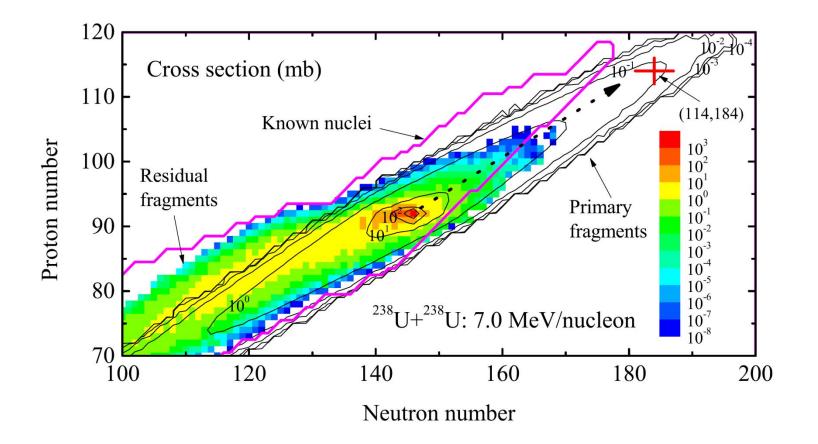
- *u*(*t*): Rel. velocity
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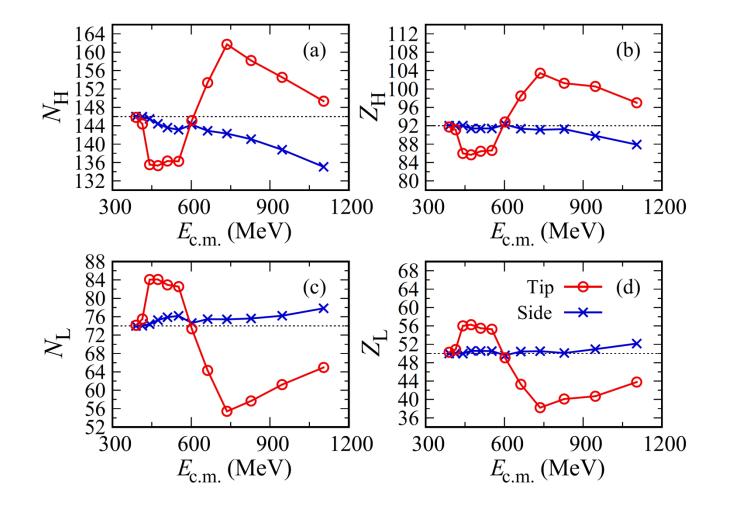


Multi-nucleon transfer reactions: ImQMD model



Zhao+2016_PRC94-024601

Multi-nucleon transfer reactions: TDHF theory



Sekizawa2017_PRC96-041601R

Lectures 3 & 4

□ Challenges in synthesizing SHN

□ Synthesis mechanism of SHN

Large uncertainties in predicted Xsections

Heavy ion fusion reactions

Capture

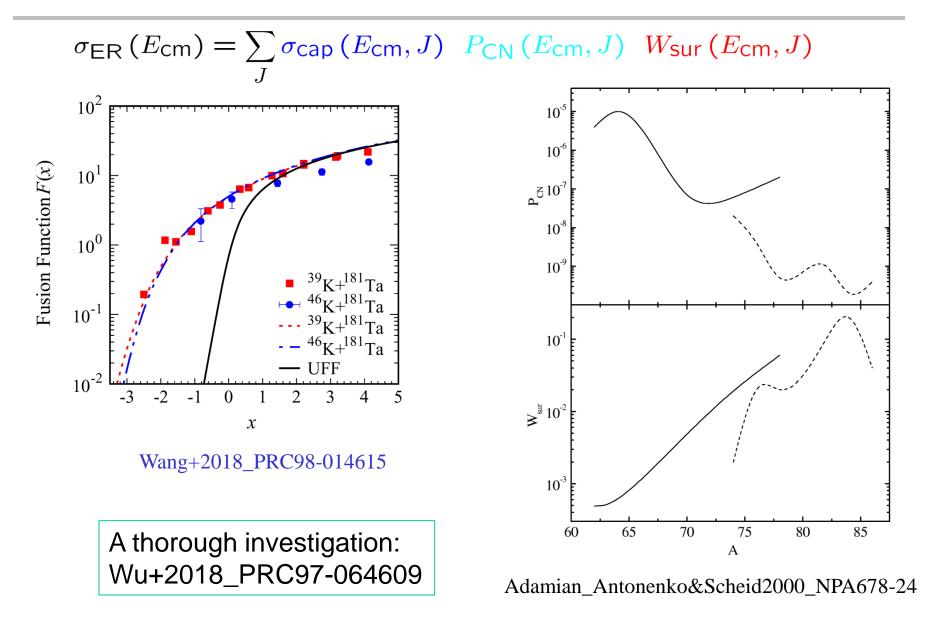
Fusion

Survival against fission

Multi-nucleon transfer reactions

Reactions w/ radioactive ion beams

Synthesis of SHN w/ radioactive ion beams



Summary

2

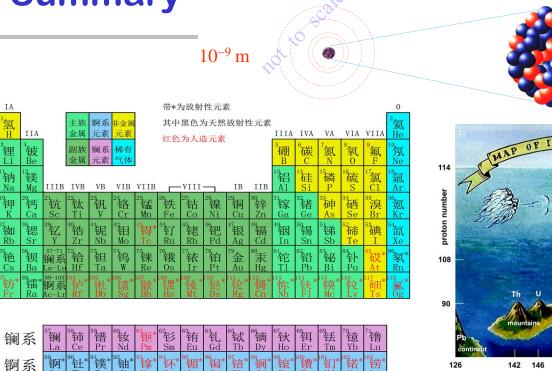
3

4

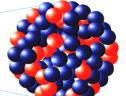
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6

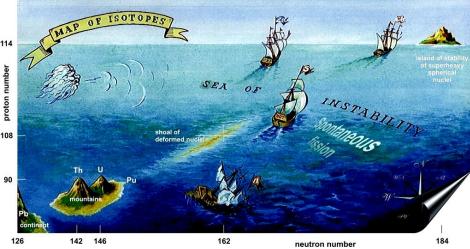
7



- □ Where is the end of PT of elements?
- Are there stable high-atomic-number elements?
- What are similarities & differences in chemistry of SHEs & their lighter homologs



 $10^{-14} \,\mathrm{m}$



- Where is the island of stability of SHN?
- □ Are there stable or long-lived SHN?
- How to reach the island of stability?
- □ Are there exotic shapes in SHN?
- Are there isomers in SHN longer-lived than their ground states?