Equation of state for neutron star using basic relativistic mean-field models

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숭실대학교 Soongsil University | OMEG INSTITUTE Origin of Matter and Evolution of Galaxies

Equation of motion

Lagrangian for free nucleon

$$\mathcal{L}_N = \Sigma_{\mathrm{N=n,p}}\, \bar{\psi}_N\left[\gamma_\mu i\partial^\mu - M_N\right]\psi_N$$

Lagrangian for meson(σ, ω)

$$\mathcal{L}_{meson} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}W_{\mu v}W^{\mu v}$$

$$\cdots W_{\mu v} = \partial_\mu\omega_v - \partial_v\omega_\mu$$

Lagrangian for nucleon-meson interaction

$$\mathcal{L}_{int} = \Sigma_{\mathrm{N=n,p}}\, \bar{\psi}_N\left[(g_\sigma\sigma) - \gamma_\mu(g_w\omega^\mu)\right]\psi_N$$

Total Lagrangian up to 4th order

Total Lagrangian

$$\mathcal{L} = \Sigma_{\mathrm{N=n,p}} \bar{\psi}_N \left[\gamma_\mu (i\partial^\mu - g_w \omega^\mu) - (M_N - g_\sigma \sigma) \right] \psi_N$$

$$+ \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} W_{\mu v} W^{\mu v} - U_{NL}(\sigma)$$

where

$$W_{\mu v} = \partial_\mu \omega_v - \partial_v \omega_\mu$$

$$U_{NL}(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

Equation of motion

Euler-Lagrange's equation

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi_\alpha)} \right] - \frac{\partial \mathcal{L}}{\partial \Psi_\alpha} = 0 \qquad (\Psi_\alpha = \psi_p, \psi_n, \overline{\psi}_p, \overline{\psi}_n, \sigma, \omega)$$

We can get the 6 kinds of the equation of motion about Ψ_α

1, 2. $\Psi_\alpha = \psi_{n,p}$

$$\bar{\psi}_{n,p} \left[i\gamma_\mu \overleftarrow{\partial}^\mu + M_N^* + g_\omega \gamma_\mu \omega^\mu \right] = 0$$

Where $M_N^* = M_N - g_\sigma \sigma$

Equation of motion

3, 4. $\Psi_\alpha = \bar{\psi}_{n,p}$

$$\left[i\gamma_\mu\partial^\mu - M_N^* - g_\omega\gamma_\mu\omega^\mu\right]\psi_{n,\,p} = 0$$

5. $\Psi_\alpha = \sigma$

$$[\partial_\mu\partial^\mu + m_\sigma^2 + g_2\sigma + g_3\sigma^2]\sigma = \Sigma_{N=n,p} g_\sigma \bar{\psi}_N \psi_N$$

6. $\Psi_\alpha = \omega^\mu$

$$\partial_\mu W^{\mu v} + m_\omega^2\omega^\mu = \Sigma_{N=n,p} g_\omega \bar{\psi}_N \gamma_\mu \psi_N$$

Where $\bar{\psi}_N\beta \to \psi^\dagger$ $\qquad \gamma^\mu = (\beta, \beta\boldsymbol{\alpha})$

Relativistic mean-field model

Energy-momentum tensor

$$T^{\mu\nu} = \sum_{\alpha} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi_\alpha)} \partial^\nu [\Psi_\alpha] - g^{\mu\nu} \mathcal{L} \qquad (\Psi_\alpha = \psi_p, \psi_n, \bar{\psi}_p, \bar{\psi}_n, \sigma, \omega)$$

Energy density

$$\mathcal{E} = \langle T^{00} \rangle$$

Pressure

$$P = \frac{1}{3} \langle T^{ii} \rangle$$

$$\langle \sigma \rangle = \bar{\sigma} + d\sigma \to \bar{\sigma}, \qquad \langle \omega_\mu \rangle = \delta_{\mu 0} (\bar{\omega} + d\omega) \to \delta_{\mu 0} \bar{\omega}$$

Equation of state

Using energy momentum tensor, we can get the equation of state

$$\mathcal{E} = \Sigma_{N=p,n} \frac{1}{\pi^2} \int_0^{K_{F_N}} dk\, k^2 \left[k^2 + {M^*}_N^2(\bar{\sigma})\right]^{1/2} + \frac{1}{2}\left(m_\sigma^2\, \bar{\sigma}^2 + m_\omega^2\, \bar{\omega}^2\right) + \frac{1}{3}\sigma^3 + \frac{1}{4}\sigma^4$$

$$P = \frac{1}{3}\Sigma_{N=p,n} \frac{1}{\pi^2} \int_0^{K_{F_N}} dk \frac{k^4}{\left[k^2 + {M^*}_N^2(\bar{\sigma}\,)\right]^{1/2}} + \frac{1}{2}\left(-m_\sigma^2\, \bar{\sigma}^2 + m_\omega^2\, \bar{\omega}^2\right) - \frac{1}{3}\sigma^3 - \frac{1}{4}\sigma^4$$

Equation of state

- Including sigma non-linear term

Binding energy give maximum value about $\rho_B = 0.148, BE \cong -16.24$ MeV

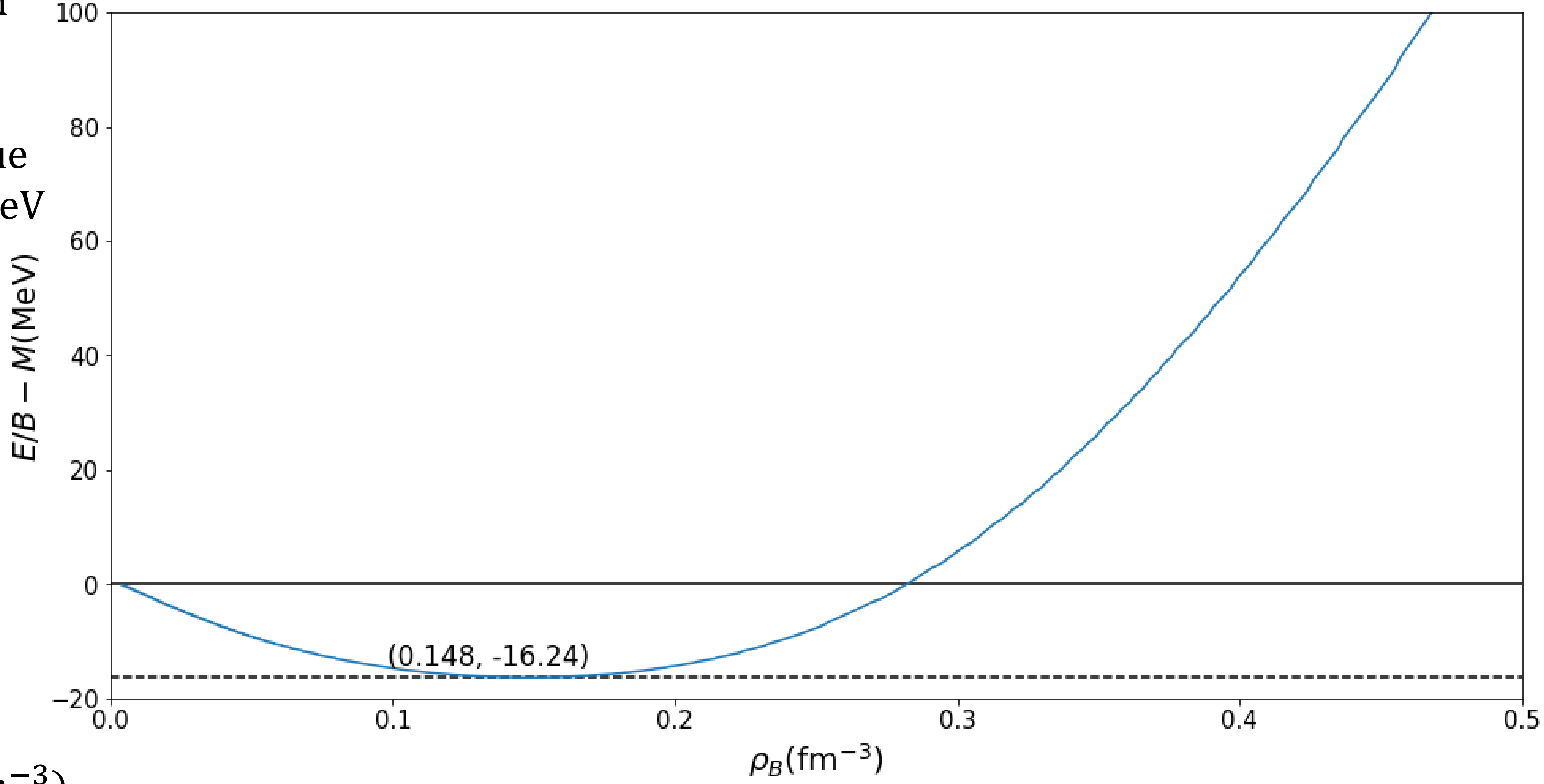
$$\frac{E}{B} = \frac{\mathcal{E}}{\rho_B}$$

(B is baryon number)

And $\rho_B = \frac{2}{3\pi^2} k_f^3$

Then

$k_f = 1.30\ (\mathrm{fm}^{-1}) \rightarrow \rho_B \cong 0.148(\mathrm{fm}^{-3})$

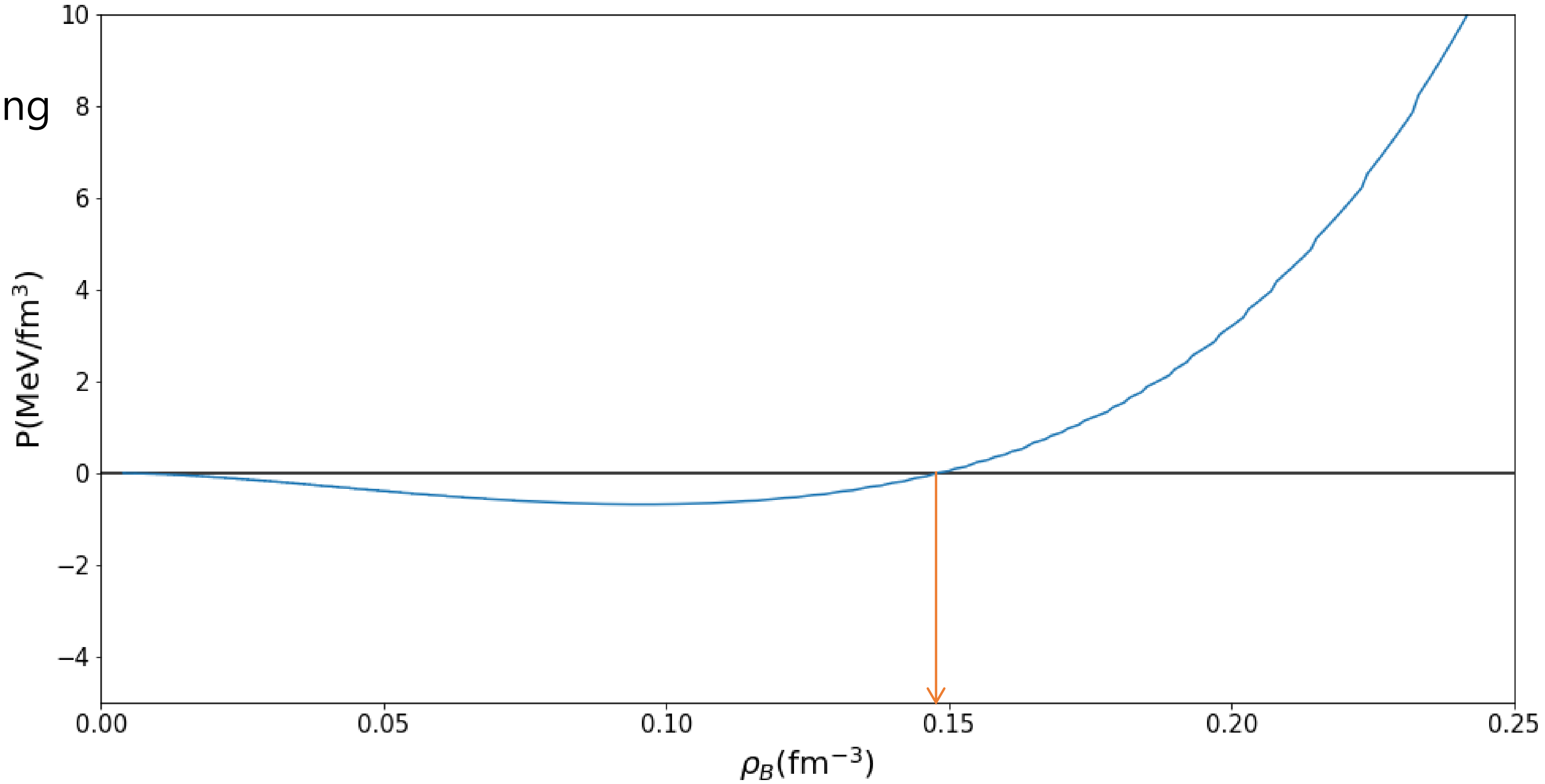


Equation of state

The pressure satisfies the following equation depending on the thermodynamic conditions.

$k_f = 1.30\ (\mathrm{fm}^{-1})$
$\rightarrow \rho_B \cong 0.148 (fm^{-3})$

$$p = \rho_B^2 \frac{\partial}{\partial \rho_B} \left(\frac{\mathcal{E}}{\rho_B}\right)$$



-SUMMARY

- Lagrangian can get from the sum of each part of Lagrangian
- Equation of motion can be derived by Euler-Lagrange's equation
- Mean-field model makes the equation simple
- If know the energy density, the saturation density point can derive
- Pressure can get the thermodynamic condition of energy density

Thank you