Moments of inertia of pairing rotation within the BCS model for Sn and Ni isotopes

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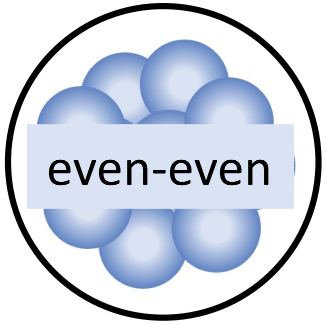
Introduction



More stable！

Due to pairing interaction

Only even-even nuclei

Ground state energy E_0

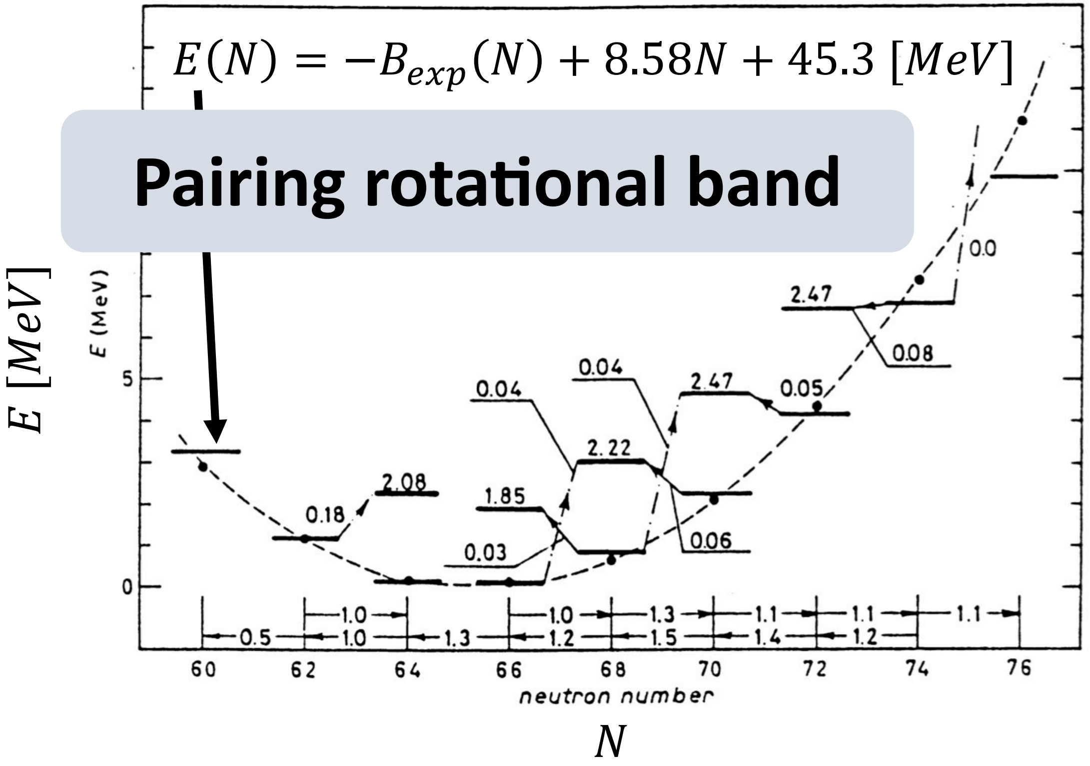
$$E_0\,(N) - E_0\,(N_0) - \frac{\partial E_0}{\partial N}(N - N_0)$$

$N_0 = 56$

$$= \frac{1}{2}\frac{\partial^2 E_0}{\partial N^2}(N - N_0)^2 \cdots$$

Second-order term of E_0 w.r.t. N

Pairing rotational energy



Pairing rotational energy in Sn isotopes

BRINK and BROGLIA , Nuclear Superfluidity (2010)

Introduction

The pair condensation in a nucleus

$\leftrightarrow$ **“deformation” in the gauge space**

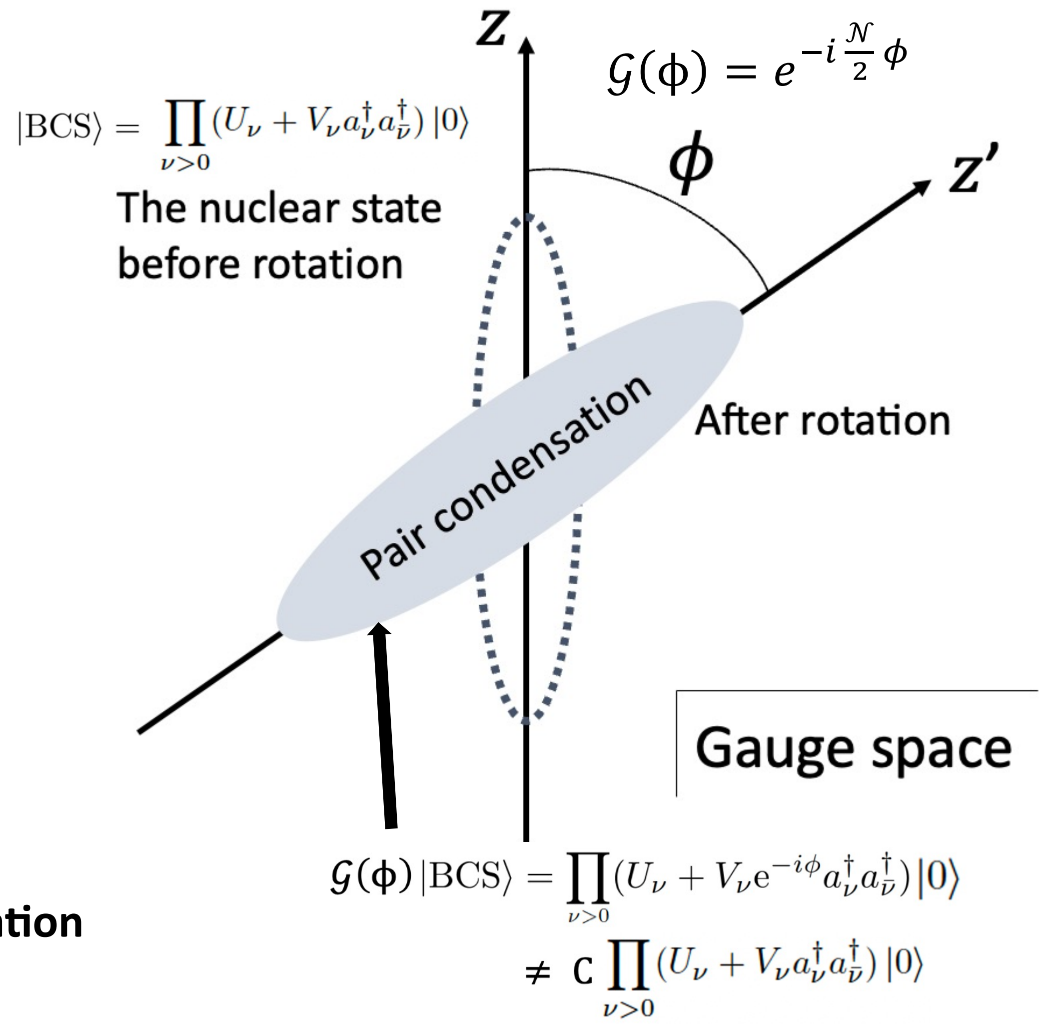
⬇

Creating a new rotational degree of freedom in the gauge space

⬇

The energy of this rotation = **pairing rotational energy**

Energy of rotation in real space		Energy of rotation in gauge space
$E(J) = \dfrac{J(J+1)}{2I}$	➡	$E(N) = \dfrac{(N-N_0)^2}{2\mathcal{J}}$
$\boldsymbol{J}$: angular momentum		N : particle number
I : moment of inertia		$\mathcal{J}$: **Moment of inertia of pairing rotation**



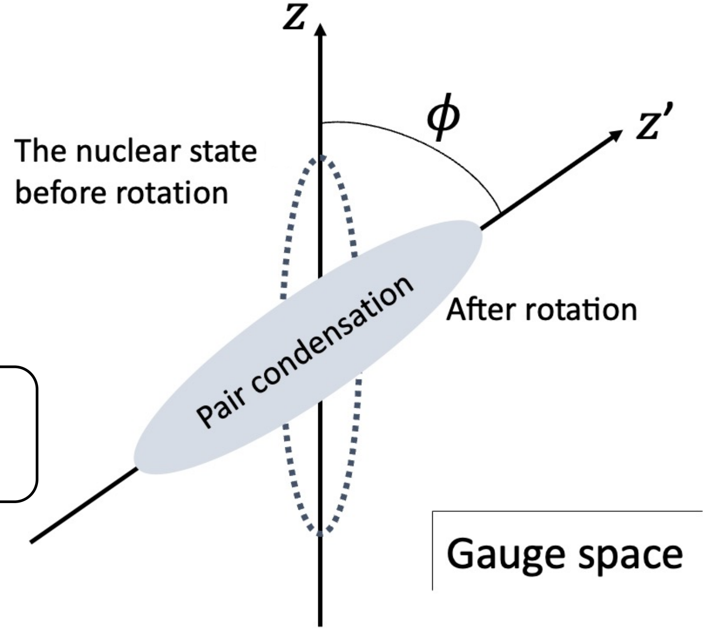
$E_0(N) = E_0(N_0) + \frac{\partial E_0}{\partial N}(N-N_0) + \frac{(N-N_0)^2}{2\mathcal{J}}$ ☜ $E(N)$ corresponds to second-order term of $E_0(N)$

Purpose

Pairing rotational energy

$$E_0(N) = E_0(N_0) + \frac{\partial E_0}{\partial N}(N - N_0) + \frac{(N - N_0)^2}{2\mathcal{J}}$$

This quantity has not received much attention



To Investigate the various properties of $\mathcal{J}$ by comparing with those of spatial rotation

Method

Hamiltonian

$$H = \sum_{\nu>0} e_\nu (a_\nu^\dagger a_\nu + a_{\bar\nu}^\dagger a_{\bar\nu}) \boxed{- G \sum_{\mu>0} \sum_{\nu>0} a_\mu^\dagger a_{\bar\mu}^\dagger a_{\bar\nu} a_\nu}$$

Pairing term

BCS model

$$|\mathrm{BCS}\rangle = \prod_{\nu>0} (U_\nu + V_\nu a_\nu^\dagger a_{\bar\nu}^\dagger) |0\rangle$$

➡ Find the ground state energy from the variational principle

$$\delta \langle \mathrm{BCS} | H - \lambda N | \mathrm{BCS} \rangle = 0 \qquad \langle \mathrm{BCS} | N | \mathrm{BCS} \rangle = n$$

☝ Condition of the number of particles

Gap Equation

$$\Delta = \frac{1}{2} G \sum_{\nu>0} \Omega_\nu \frac{\Delta}{\sqrt{(e_\nu - \lambda)^2 + \Delta^2}}$$

☜ Calculate self-consistently

where $\Delta = G \sum_{\nu>0} \Omega_\nu U_\nu V_\nu$

➡ Determine U, V, and calculate ground state energy E_0

$$E_0 = 2 \sum_\nu \Omega_\nu e_\nu V_\nu^2 - \frac{\Delta^2}{G} - G \sum_\nu \Omega_\nu V_\nu^4$$

➡ Calculate **MOI of pairing rotation**

$$\mathcal{J} = \frac{1}{\frac{\partial^2 E_0}{\partial N^2}} = \frac{4}{E_0(N-2) - 2E_0(N) + E_0(N+2)}$$

$$\begin{aligned} \Delta S_{2n}(N) &= S_{2n}(N) - S_{2n}(N+2) \\ &= E_0(N-2) - 2E_0(N) + E_0(N+2) \end{aligned}$$

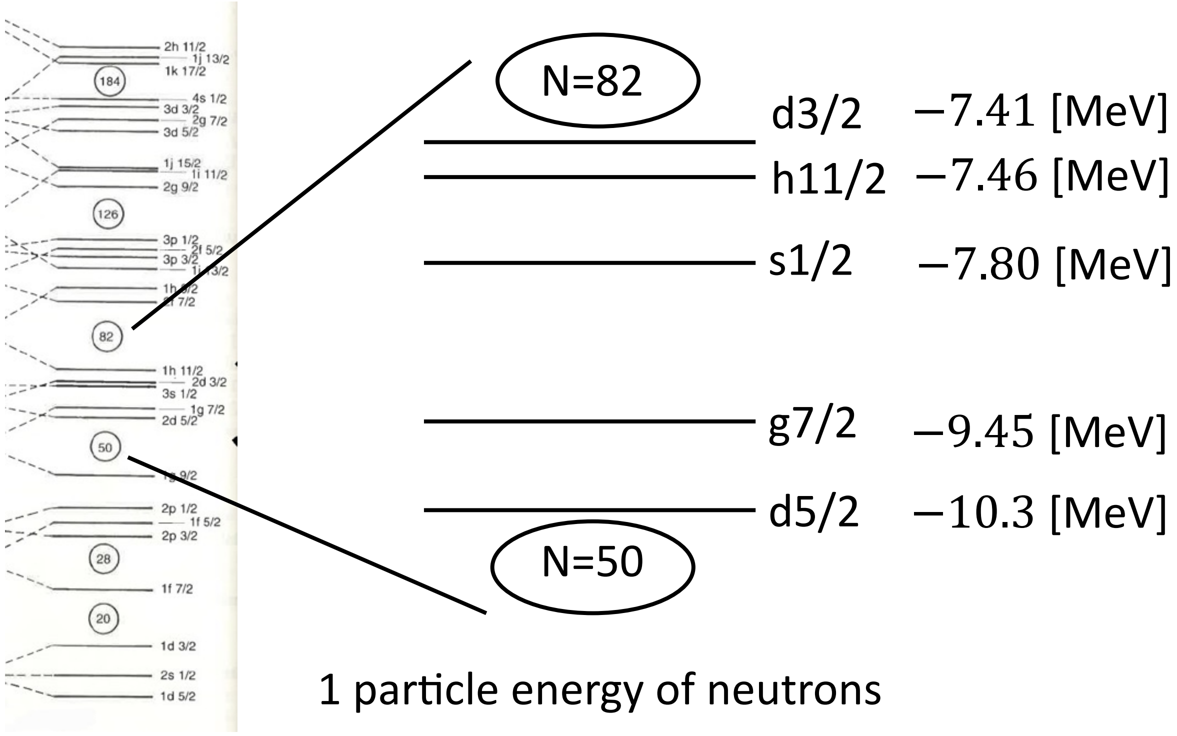
➡ $\Delta S_{2n} = 4/\mathcal{J}$

Method

Only neutron pairing is considered

Sn

$Z = 50$ (magic)
$N = 52\sim80$

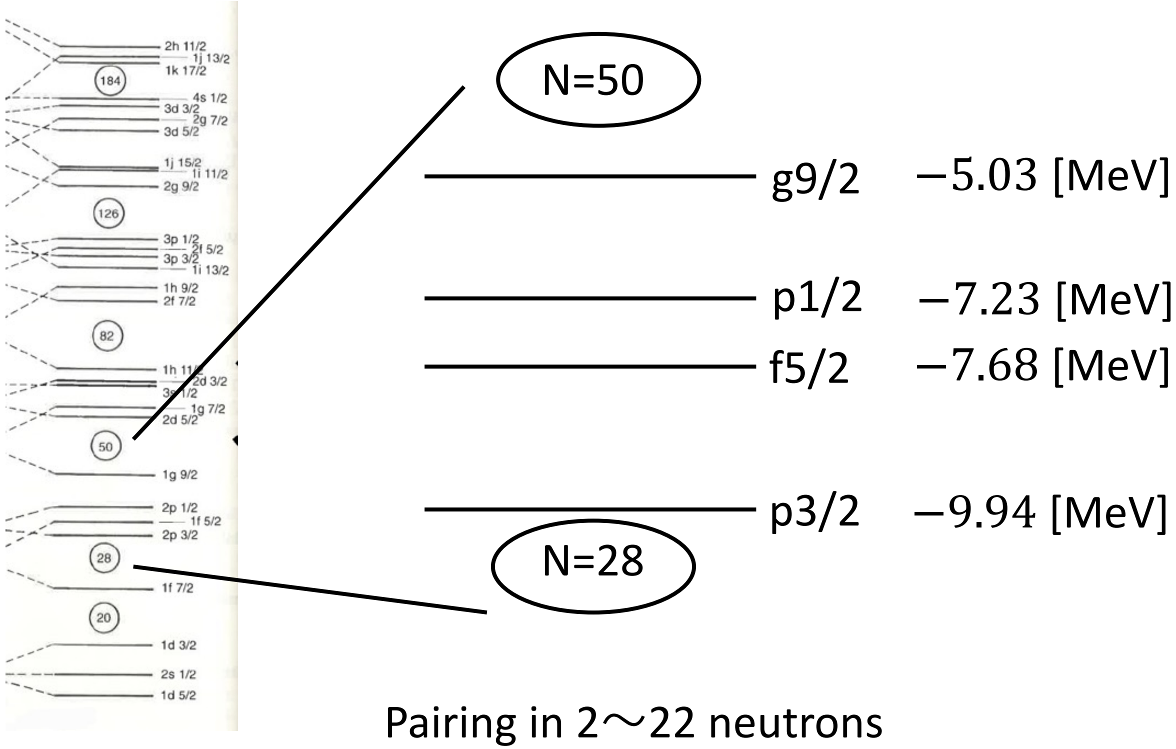


1 particle energy of neutrons

Pairing in 2～30 neutrons
in valence space is included

Ni

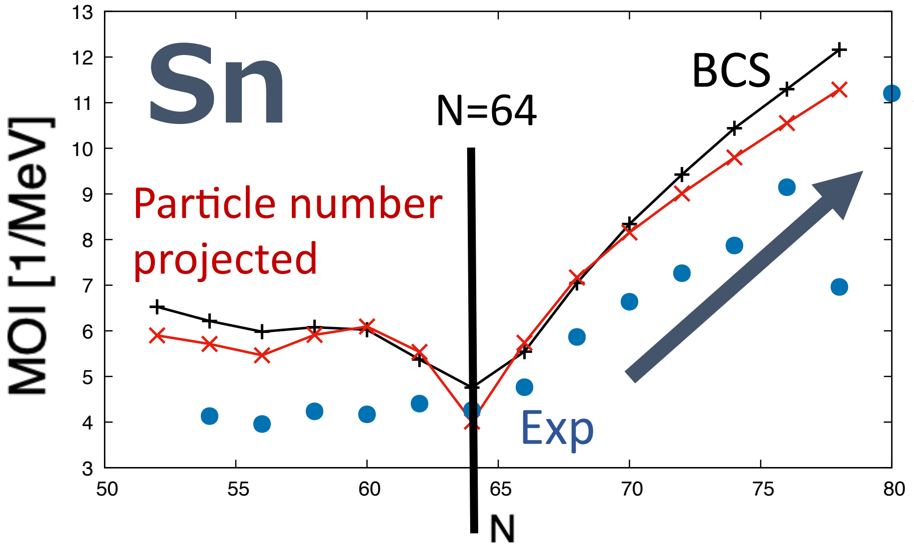
$Z = 28$ (magic)
$N = 30\sim48$



Pairing in 2～22 neutrons
in valence space is included

S. Nilsson, I. Ragnarsson, Shapes and shells in nuclear structure (1995)

N dependence of MOI



N increases ➡ MOI increases

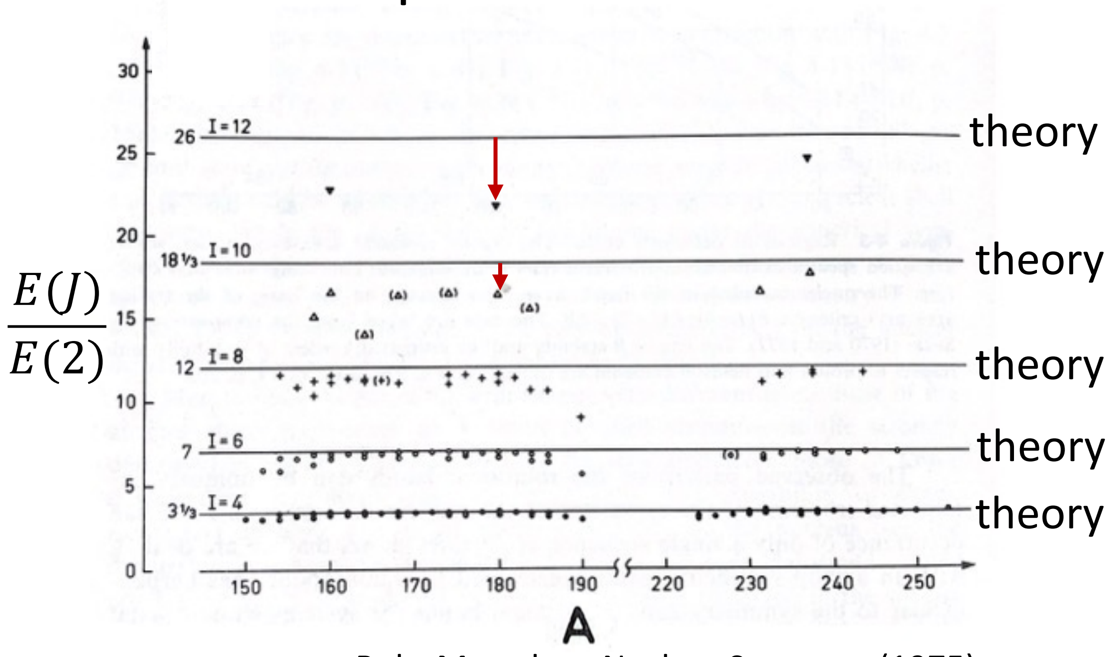
☜ Same as the relationship between J--MOI in spatial

Particle number projected :

$$E_{0(\text{projected})} = \langle n | H | n \rangle$$

☝ Particle number eigen state (after projected)

Spatial rotation

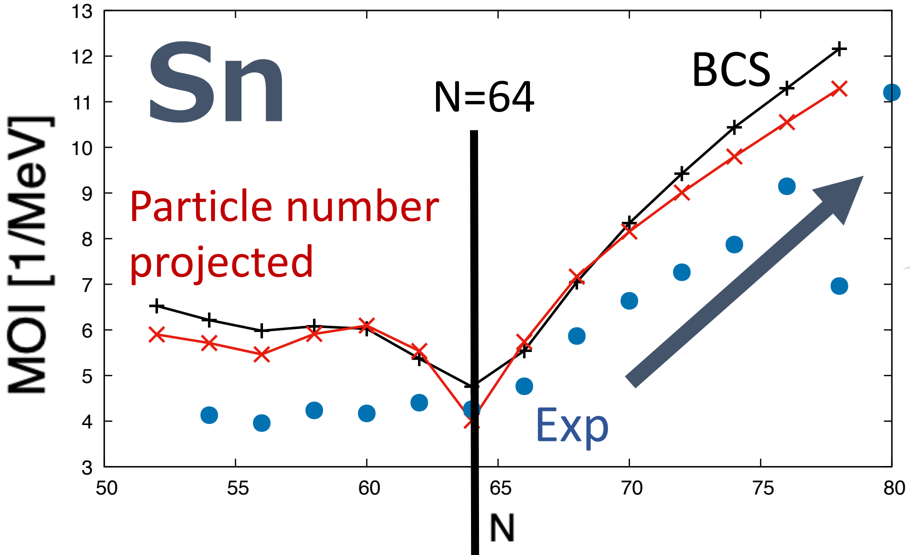


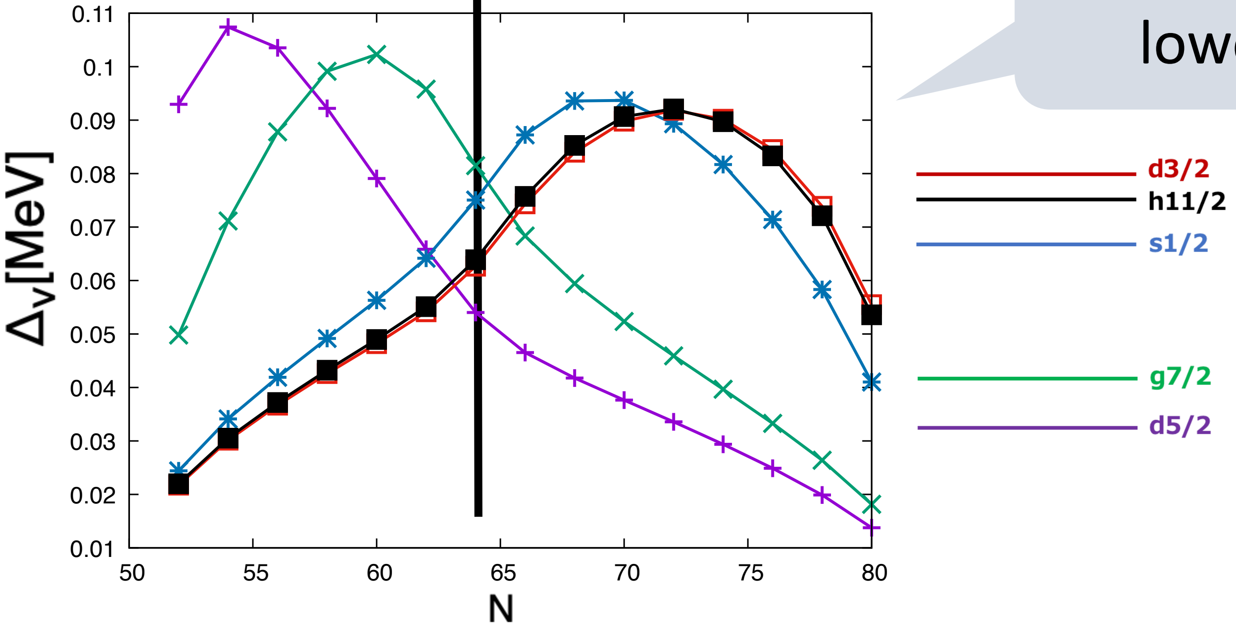
Bohr Mottelson Nuclear Structure (1975)

$$E(J) = \frac{J(J+1)}{2I}$$

J increases ➡ MOI increases

N dependence of MOI





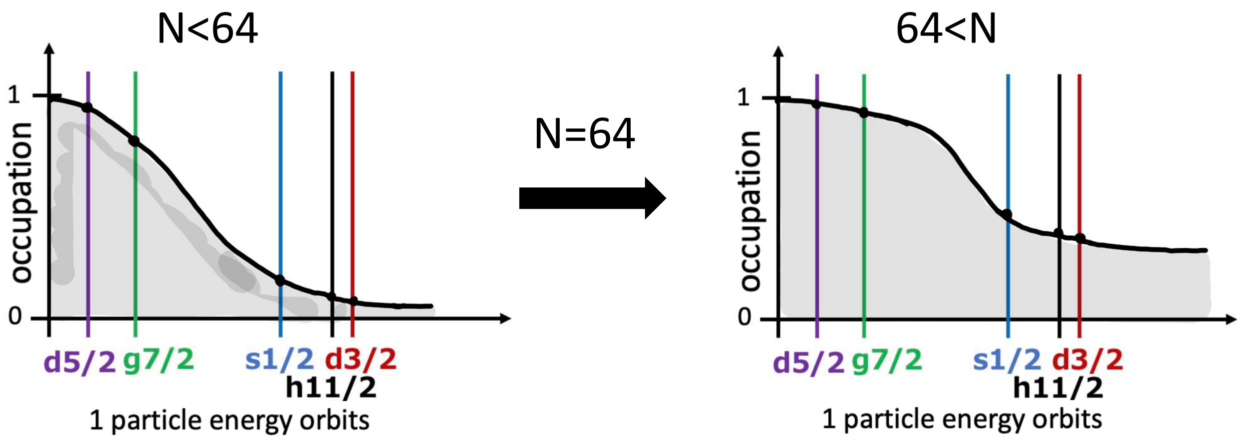
Why?

Above N=64, the increasing trend of MOI changes

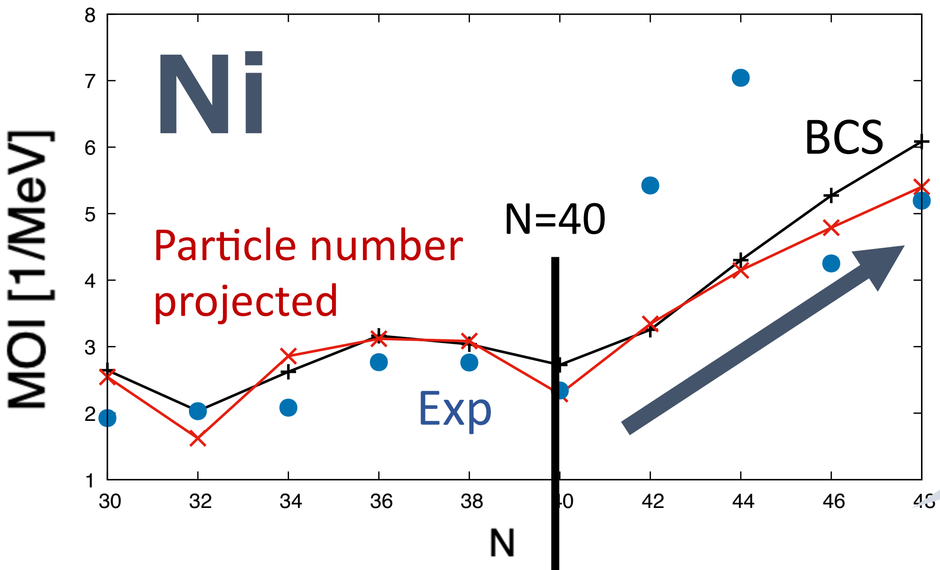
$$\Delta = \sum_{\nu>0} \Omega_\nu \Delta_\nu$$

Pairing gap of each orbit

The neutron contributing to superfluidity moves from lower 2 energy orbits to higher 3 energy orbits



N dependence of MOI



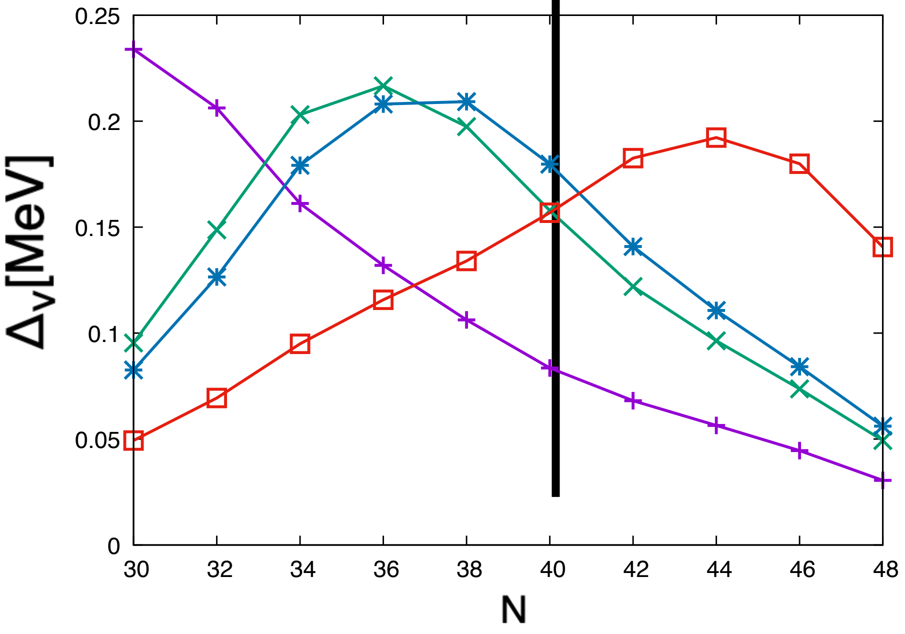
- N increases ➡ MOI increases
- After N=40, the increasing trend of MOI changes

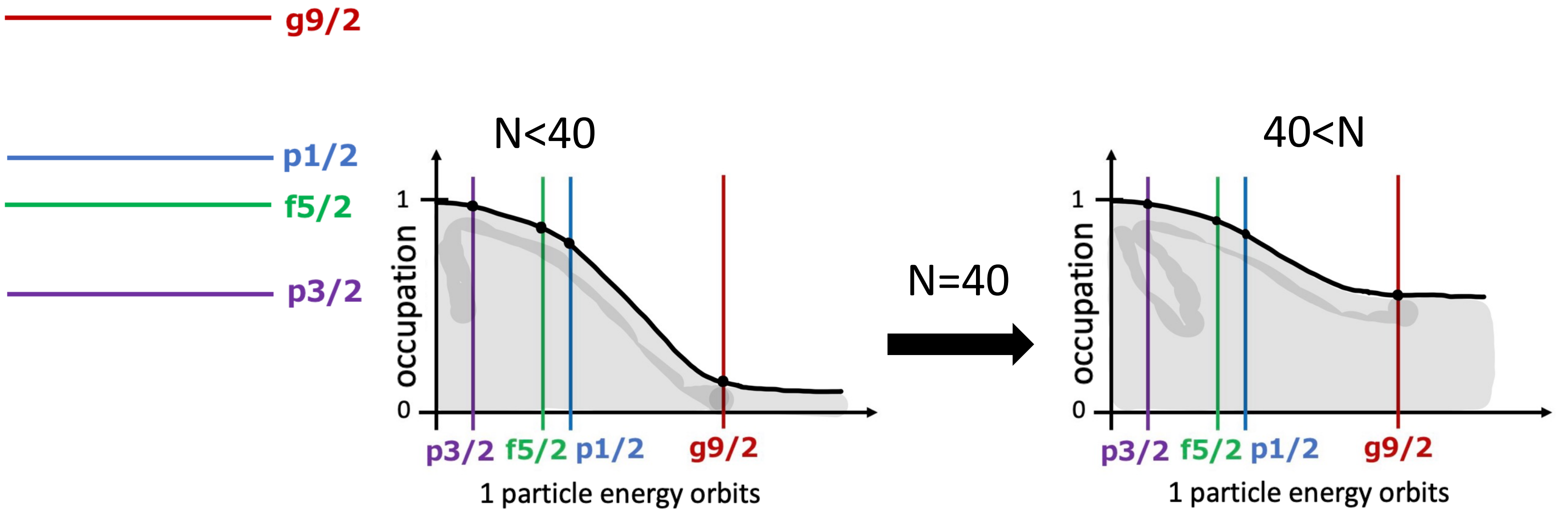
Same as Sn !

The neutron contributing to superfluidity moves from lower 3 energy orbits to higher 1 energy orbits

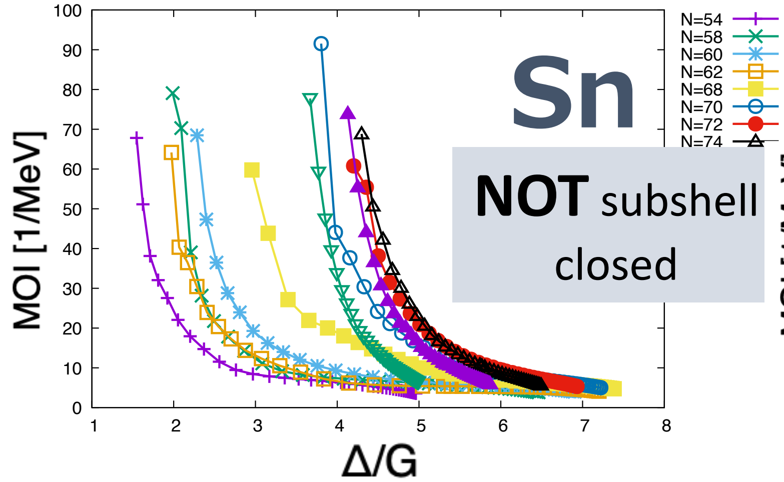
$$\Delta = \sum_{\nu>0} \Omega_\nu \Delta_\nu$$

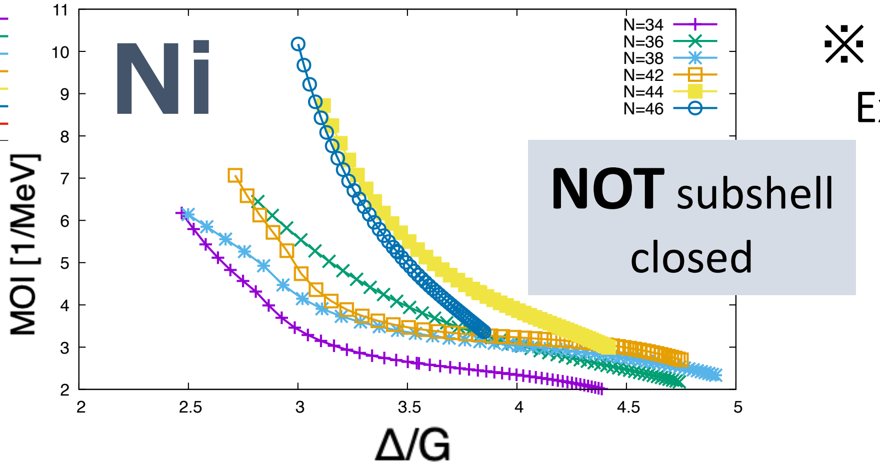
Pairing gap of each orbit

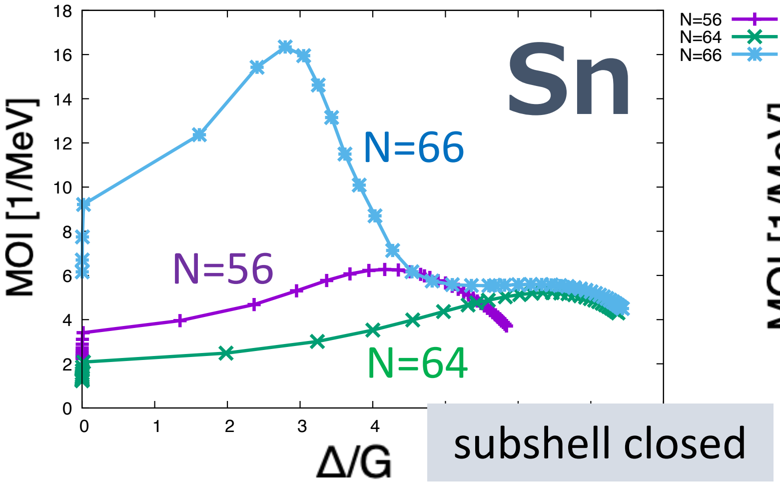


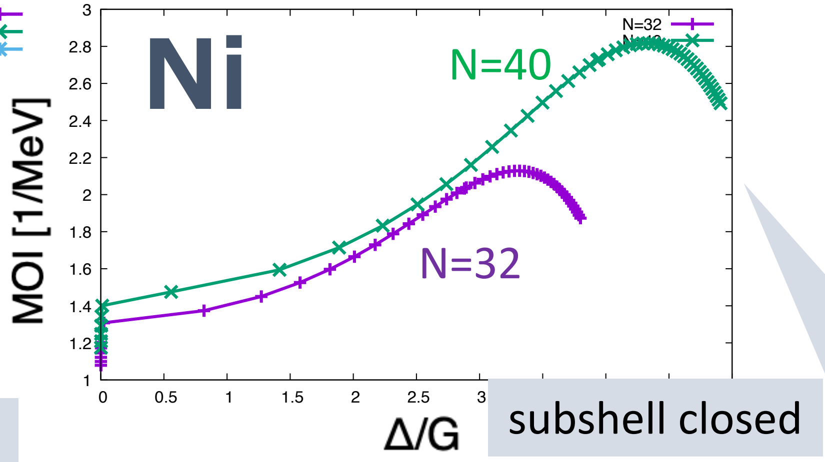


Δ dependence of MOI



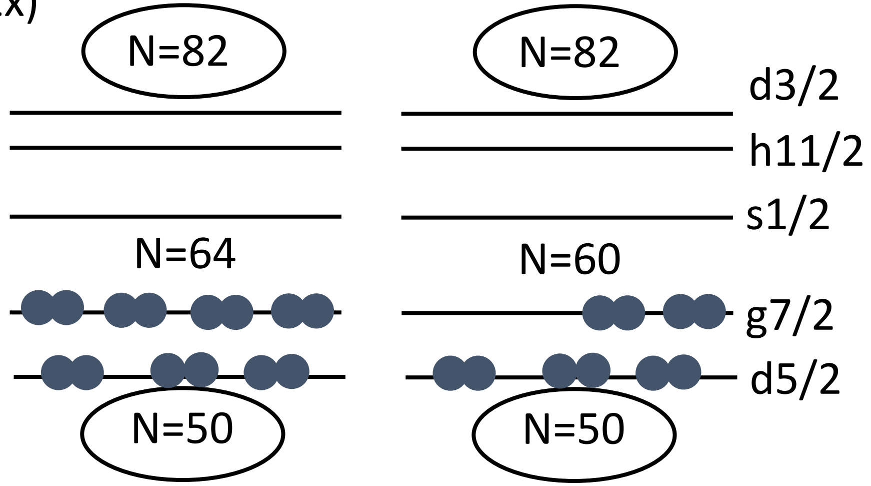






※ When pairing strength $G\sim 0$ (NO pairing)

Ex)



Subshell closed | **NOT** Subshell closed

Sn: N=54～78 **Ni**: N=34～46

$$\Delta/G = \sum_{\nu} \Omega_\nu U_\nu V_\nu$$

Δ increase ➡ MOI decrease

(except subshell closed nuclei)

↕

Different from spatial rotation

MOI ∝ β^2

Summary & Future plans

We investigate the property of MOI of pairing rotation using BCS theory.

- In both Sn and Ni, the relationship between particle number and the MOI of pairing rotation is **roughly consistent** with that between angular momentum and MOI of **spatial rotation**.
- Particle number dependence of MOI change at $^{114}\mathrm{Sn}$ and $^{68}\mathrm{Ni}$, because the orbit **most contributing to pairing collation change** into a higher energy.

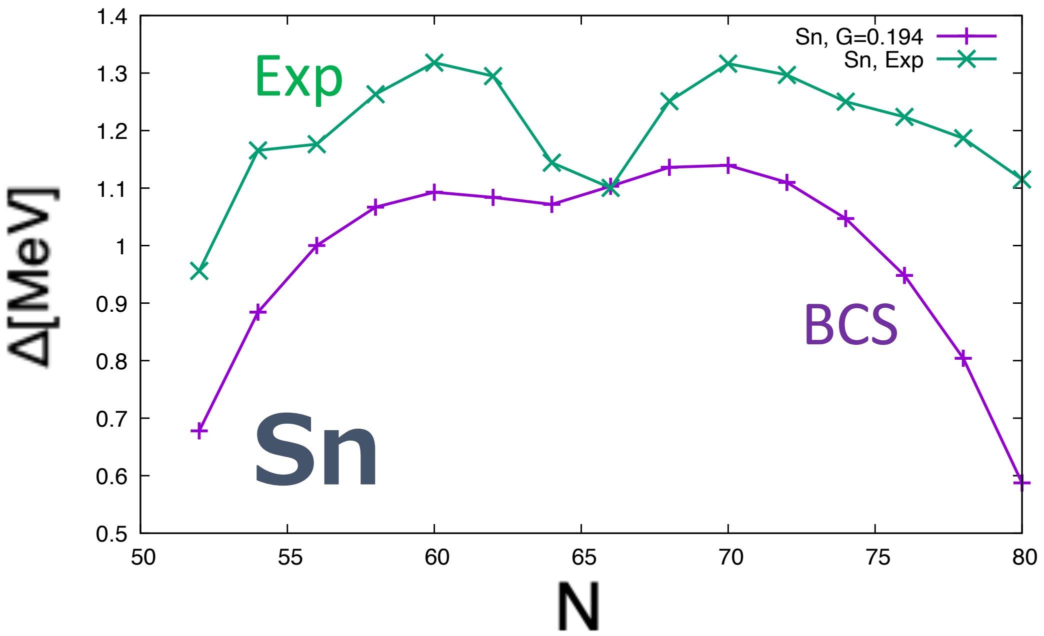
- The "**deformation" dependence** of the MOI of pairing rotation is **very different** from that of **spatial rotation**.
- The "deformation" dependence of MOI of pairing rotation in the nuclei that are in the **subshell-closed nuclei** is different from that of in **open-shell nuclei** .

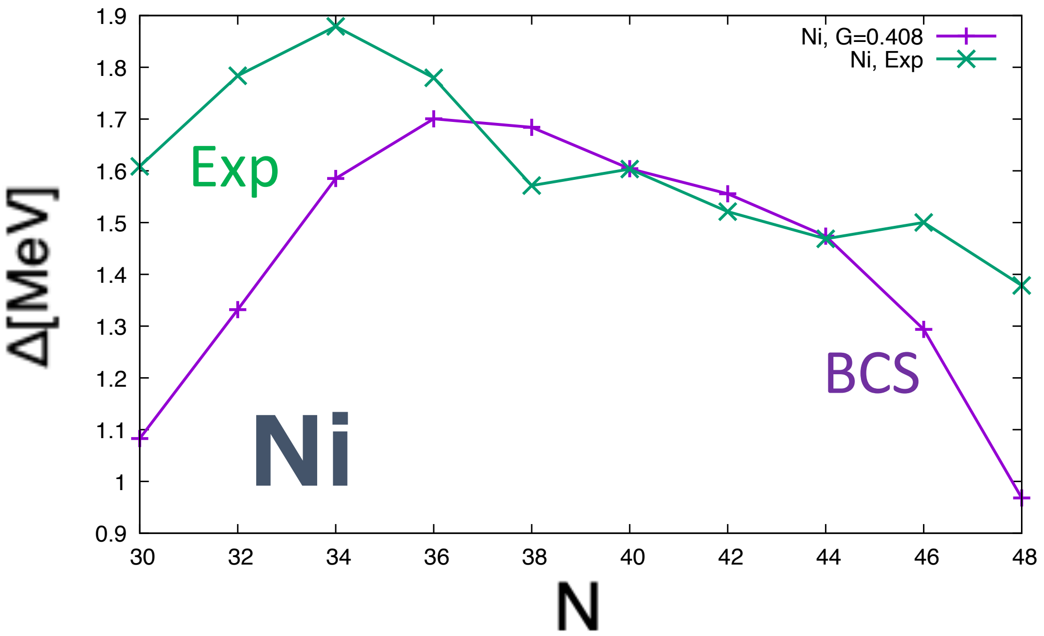
➢ I want to analyze the change of inner structure with respect to particle number to investigate whether there is a phenomenon that is **similar to "back bending"** in spatial rotation.

➢ I will check whether the same results are produced by the **HFB** and the shell model code "**KSHELL**". ☜ now doing !

KSHELL : https://sites.google.com/alumni.tsukuba.ac.jp/kshell-nuclear/ created by Noritaka Shimizu

Supplement : N-Pairing gapΔ



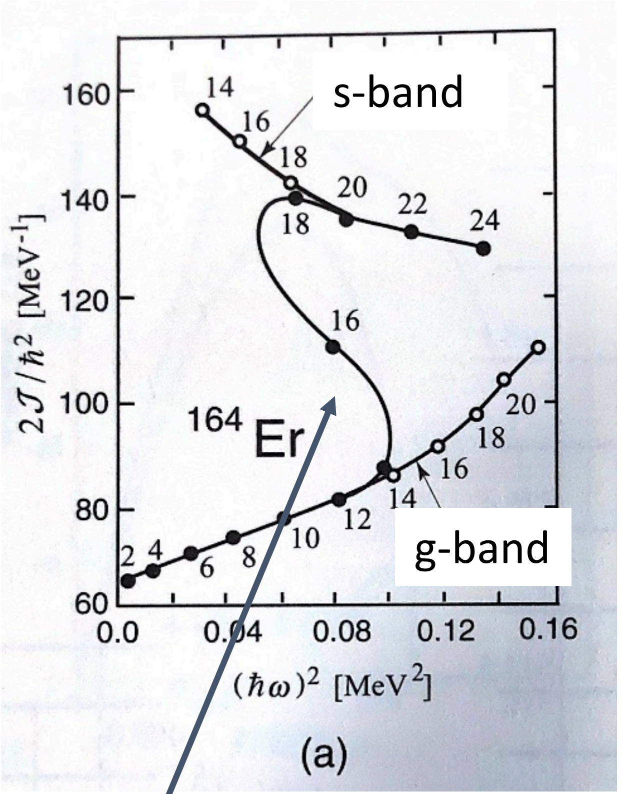


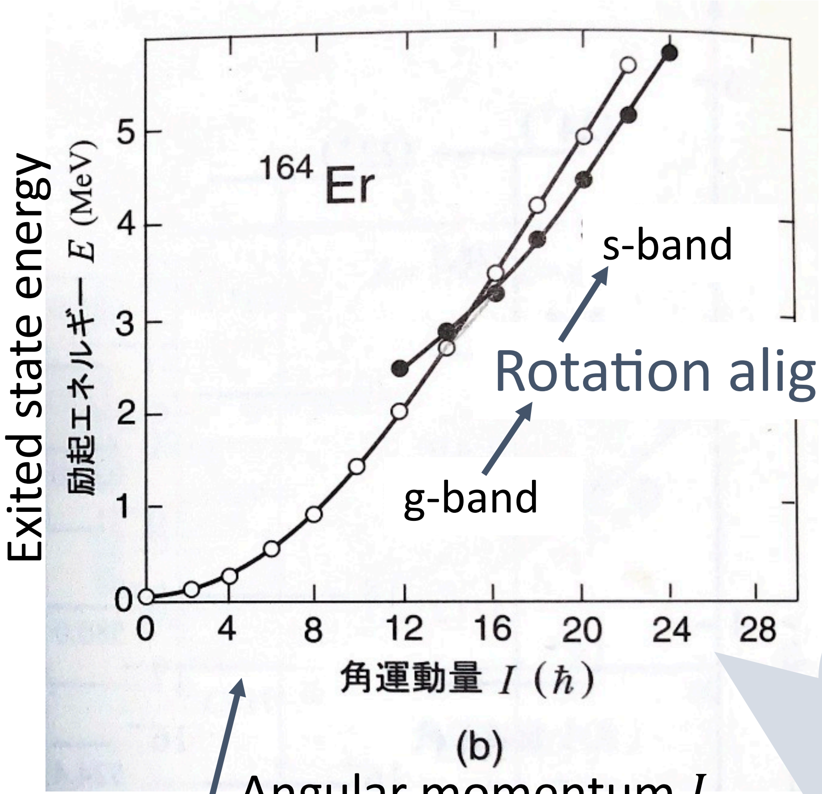
Pairing gap : $\Delta = G \sum_{\nu>0} \Omega_\nu \Delta_\nu$

$= G \sum_{\nu>0} \Omega_\nu \langle a_\nu a_{\bar{\nu}} \rangle$

☜ Energy because of pairing
Something like “deformation”

Supplement : Back-bending





Back-bending

Rotation alignment

Band crossing

Angular momentum I

Takada,Ikeda 原子核構造論

Back-bending :

MOI suddenly increases at angular momentum I=14,16 with respect to angular velocity ω

because

Band-crossing :

Rotational energy jump from g-band to s-band

Whether there is a similar phenomenon in pairing rotation? (N-MOI)