Neutron Capture Reactions and Nucleosynthesis in Astrophysics

QRPA calculations for M1 transitions and the application to the neutron radiative capture cross sections

Hirokazu Sasaki, Toshihiko Kawano, Ionel Stetcu

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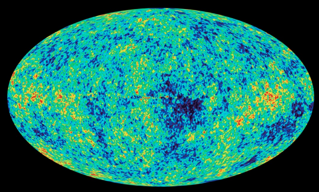
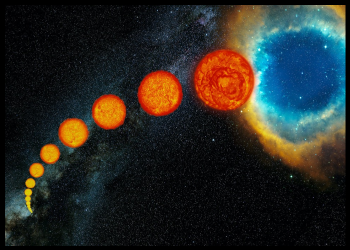
Nucleosynthesis in the Universe

Early Universe



Big Bang
Nucleosynthesis

Inside stars



s-process

α-process

CNO cycle

pp chain ...

Explosive phenomena

r-process

γ-process

νp-process ...

Theoretical nuclear physics and astrophysics help us understand the origin of elements in the Universe

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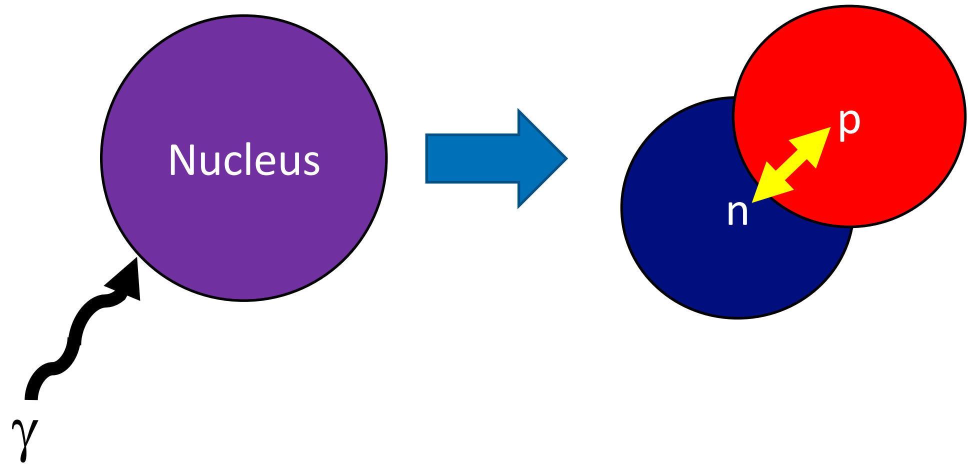
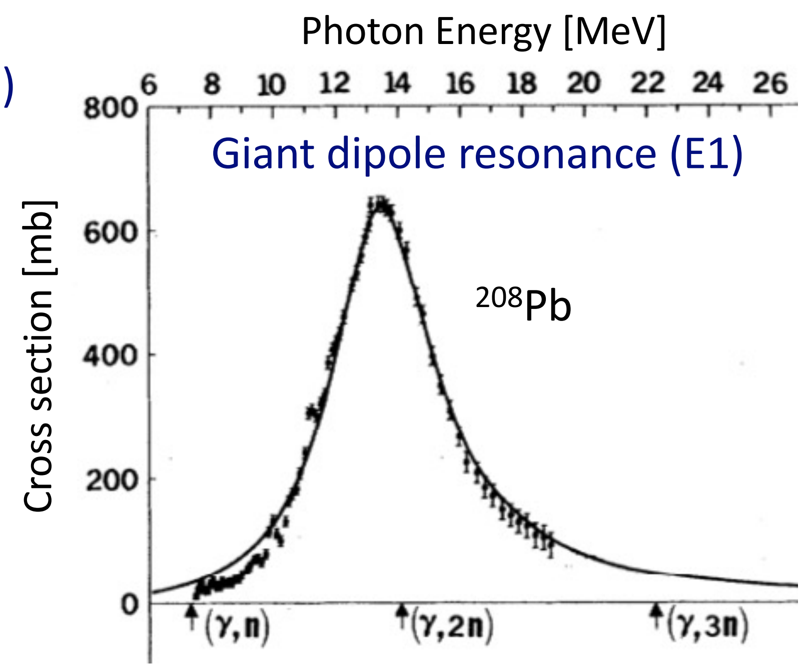
Giant resonance (GR)

GR is collective motions of many neutrons and protons inside the nucleus

GR is characterized by various oscillation modes of nucleons (E0, E1,M1,E2,M2..)





B.L.Berman and S.C.Fultz
Rev. Mod. Phys.47,3 (1975)

Information on the nuclear structure
(e.g., Symmetry energy, compressibility, …)

Applications
(e.g., Nuclear reaction theory, data evaluation, nucleosynthesis in the Universe, …)

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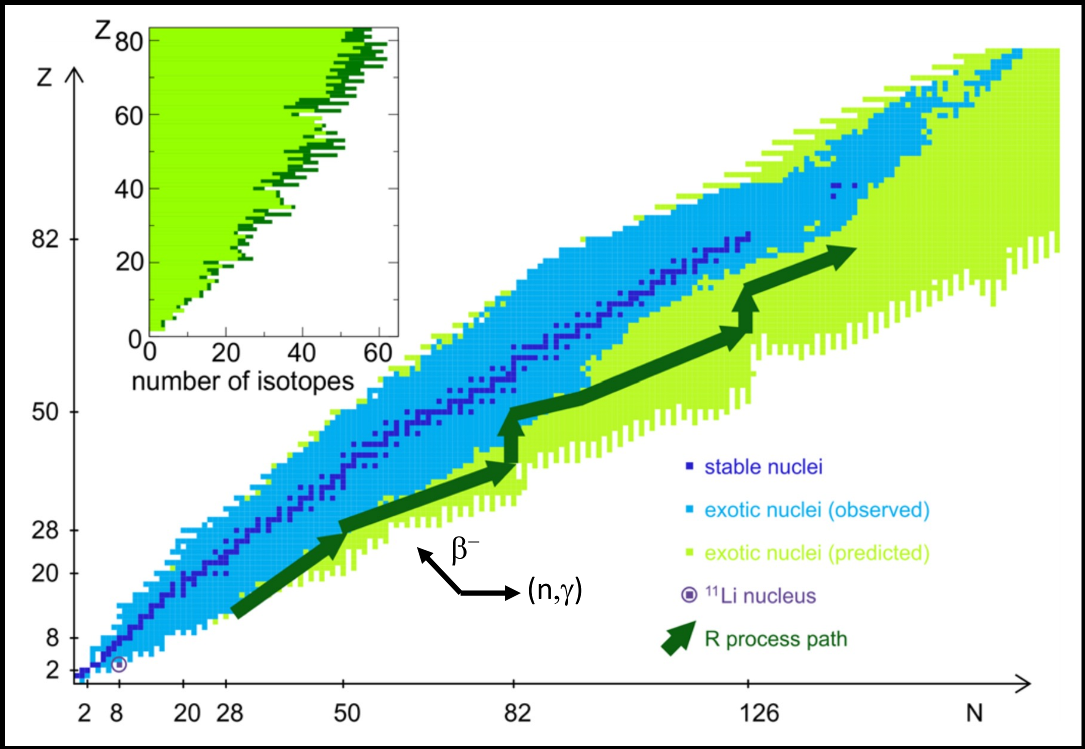
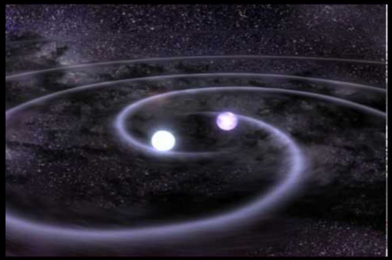
Rapid neutron capture process (r-process)

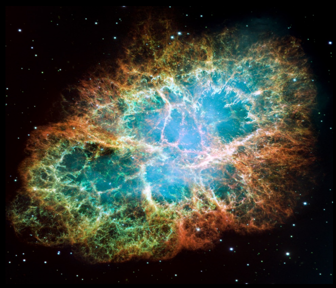
Possible astrophysical sites



T. Otsuka et al., Rev.Mod.Phys.92, 015002(2020)



Neutron star mergers



Core-collapse SNe
(MHDSNe, Collapsars)

The r-process is a key to reveal the origin of heavy elements in the Universe

Theoretical GR calculations are needed for (n,γ) reactions on unstable nuclei

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How to calculate GR cross sections?

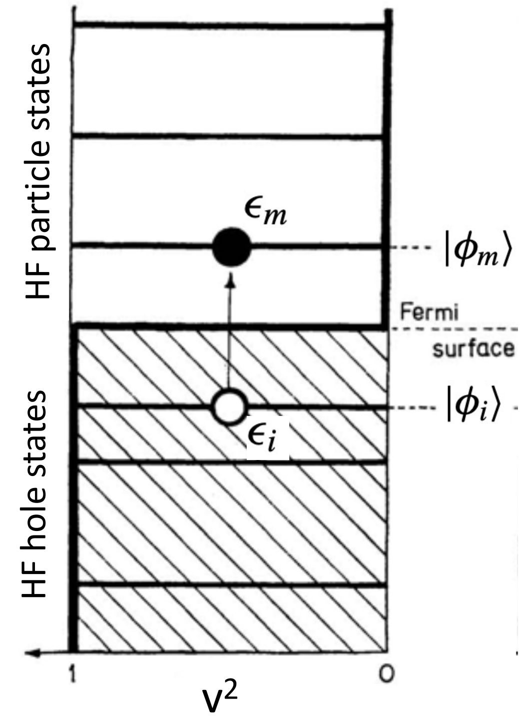
1. The linear response of the time-dependent Hartree Fock (HF) equations

$$(\epsilon_m - \epsilon_i - \omega)X_{mi}(\omega) + \langle\phi_m|\,\delta h(\omega)\,|\phi_i\rangle = -\langle\phi_m|\,V_{\rm ext}(\omega)\,|\phi_i\rangle$$

$$(\epsilon_m - \epsilon_i + \omega)Y_{mi}(\omega) + \langle\phi_i|\,\delta h(\omega)\,|\phi_m\rangle = -\langle\phi_i|\,V_{\rm ext}(\omega)\,|\phi_m\rangle$$

$V_{\rm ext}(\omega)$: Weak external field

$\delta h(\omega)$: Residual interaction



2. Transition strength

$$\frac{dB(E;V_{\rm ext})}{dE} = -\frac{1}{\pi}{\rm Im}\sum_q \sum_{m,i\in q}\left(f_{mi}^{q*}X_{mi}^q + f_{im}^{q*}Y_{mi}^q\right)$$

$$f_{mi}^q = \int d^3r\,\phi_m^{q*}V_{\rm ext}\phi_i^q \quad q \text{ ... n, p}$$

3. Cross section

e.g.) $V_{\rm ext} = \sum_{K=0,\pm1} D_K$

$$\sigma_{\rm abs}(E;{\rm E1}) = \frac{16\pi^3}{9\hbar c}E\sum_{K=0,\pm1}\frac{dB(E;D_K)}{dE}$$

E1 operator

$$D_K = \sum_{i=1}^{A} e_{\rm eff}^{(i)} r_i Y_{1K}(\theta_i,\varphi_i)$$

Finite amplitude method (FAM)

FAM is a calculation method for the residual interaction

$$\delta h(\omega) = \frac{1}{\eta}(h[\langle\psi'|, |\psi\rangle] - h[\langle\phi|, |\phi\rangle])$$

η : Small parameter

$$\langle\psi_i'| = \langle\phi_i| + \eta\langle Y_i(\omega)|$$

$$|\psi_i\rangle = |\phi_i\rangle + \eta|X_i(\omega)\rangle$$

T. Nakatsukasa et al., PRC76, 024318(2007)

Application

・Multipole modes

M. Kortelainen et al., PRC92, 051302(R)(2015), T. Oishi et al., PRC93, 034329(2016)

・β decays

E.M. Ney et al., PRC102, 034326(2020), N. Hinohara and J. Engel, PRC105, 044314(2022)

・Fission dynamics

K. Washiyama et al., PRC103, 014306(2021)

RPA equation in noniterative FAM

$$(\epsilon_m - \epsilon_i - \omega) X_{mi}(\omega) + \langle \phi_m | \delta h(\omega) | \phi_i \rangle = - \langle \phi_m | V_{\rm ext}(\omega) | \phi_i \rangle$$

$$(\epsilon_m - \epsilon_i + \omega) Y_{mi}(\omega) + \langle \phi_i | \delta h(\omega) | \phi_m \rangle = - \langle \phi_i | V_{\rm ext}(\omega) | \phi_m \rangle$$

Explicit linearization

$$\lim_{\eta \to 0} \delta h = \sum_{q'} \sum_{nj \in q'} X_{nj}^{q'} \left. \frac{\partial h}{\partial \left(\eta X_{nj}^{q'}\right)} \right|_{\eta=0} + \sum_{q'} \sum_{nj \in q'} Y_{nj}^{q'} \left. \frac{\partial h}{\partial \left(\eta Y_{nj}^{q'}\right)} \right|_{\eta=0}.$$

RPA equation

$$\left\{ \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \begin{pmatrix} X_{nj}^{q'} \\ Y_{nj}^{q'} \end{pmatrix} = - \begin{pmatrix} f_{mi}^{q} \\ f_{im}^{q} \end{pmatrix}$$

$$A_{mi,nj}^{q,q'} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \int d^3 r \, \phi_m^{q*} \left(\frac{\partial h_q}{\partial \left(\eta X_{nj}^{q'}\right)} \right)_{\eta=0} \phi_i^q \qquad f_{mi}^q = \int d^3 r \, \phi_m^{q*} V_{\rm ext} \phi_i^q$$

$$B_{mi,nj}^{q,q'} = \int d^3 r \, \phi_m^{q*} \left(\frac{\partial h_q}{\partial \left(\eta Y_{nj}^{q'}\right)} \right)_{\eta=0} \phi_i^q$$

We can avoid the iteration in conventional FAM

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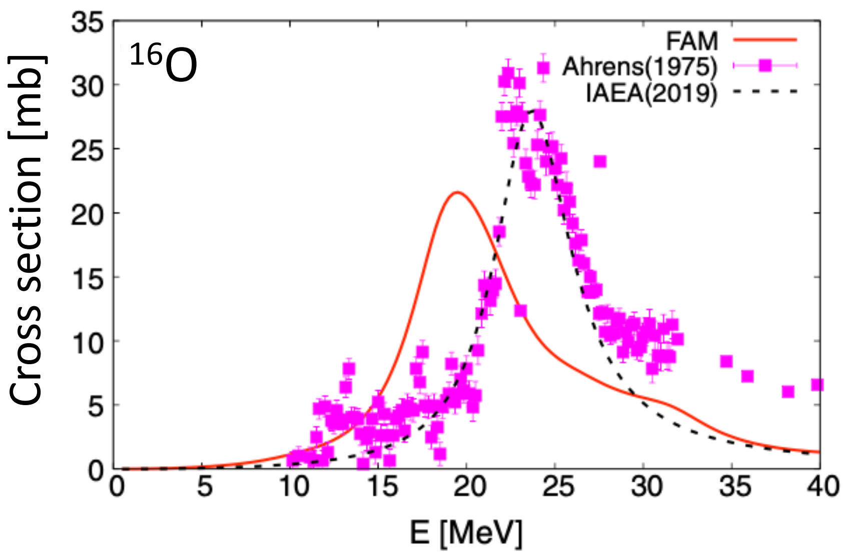
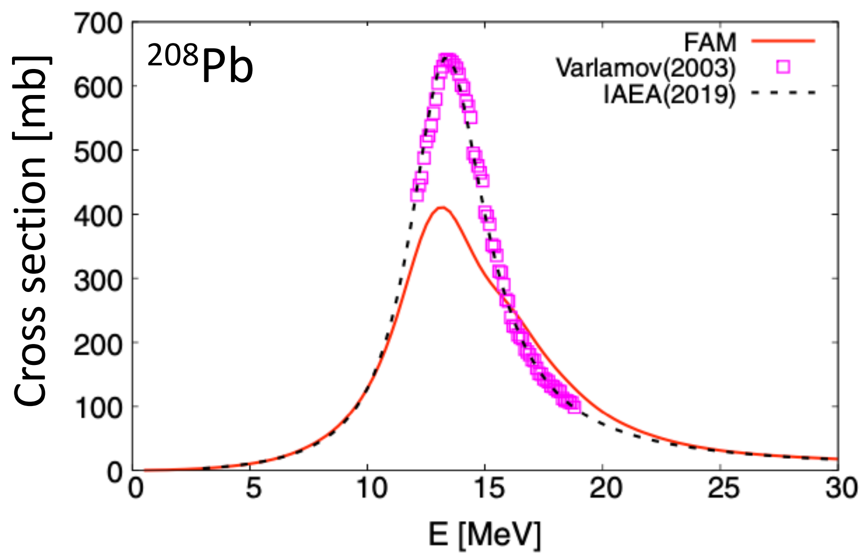
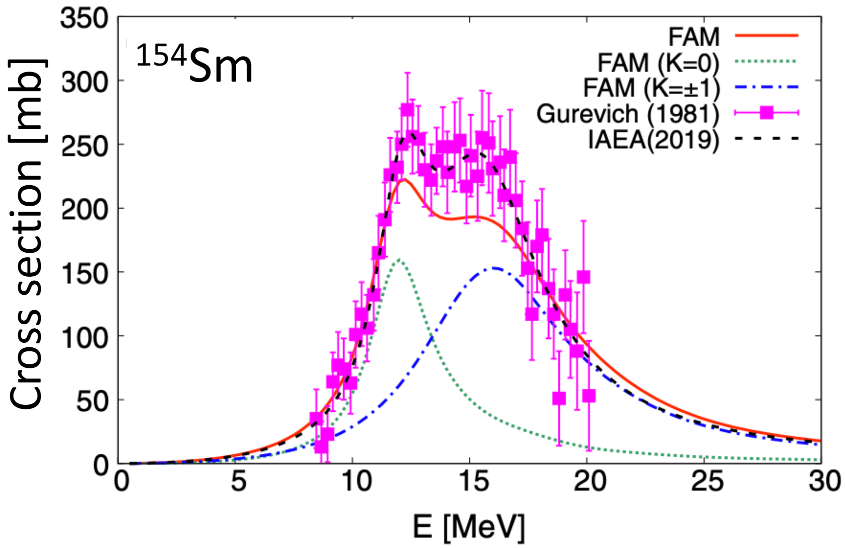
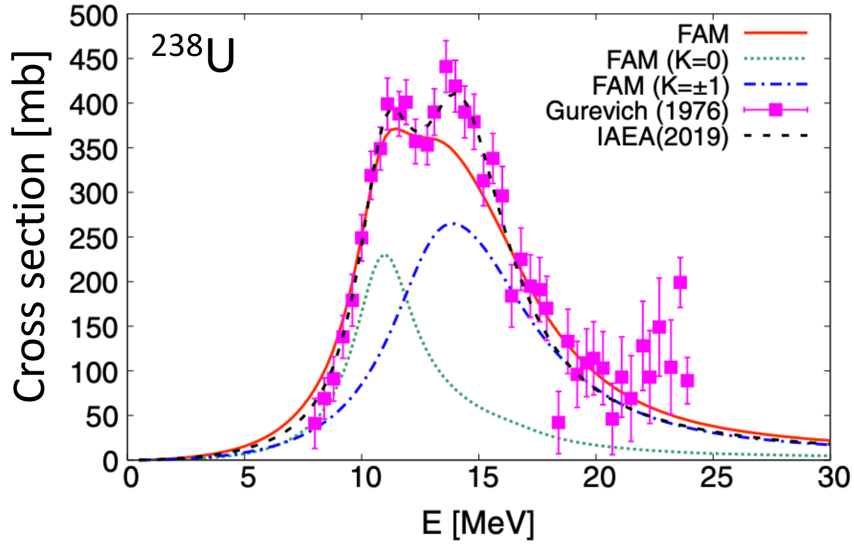
Our FAM results for giant dipole resonances

H. Sasaki, T. Kawano and I. Stetcu, PRC105, 044311(2022)









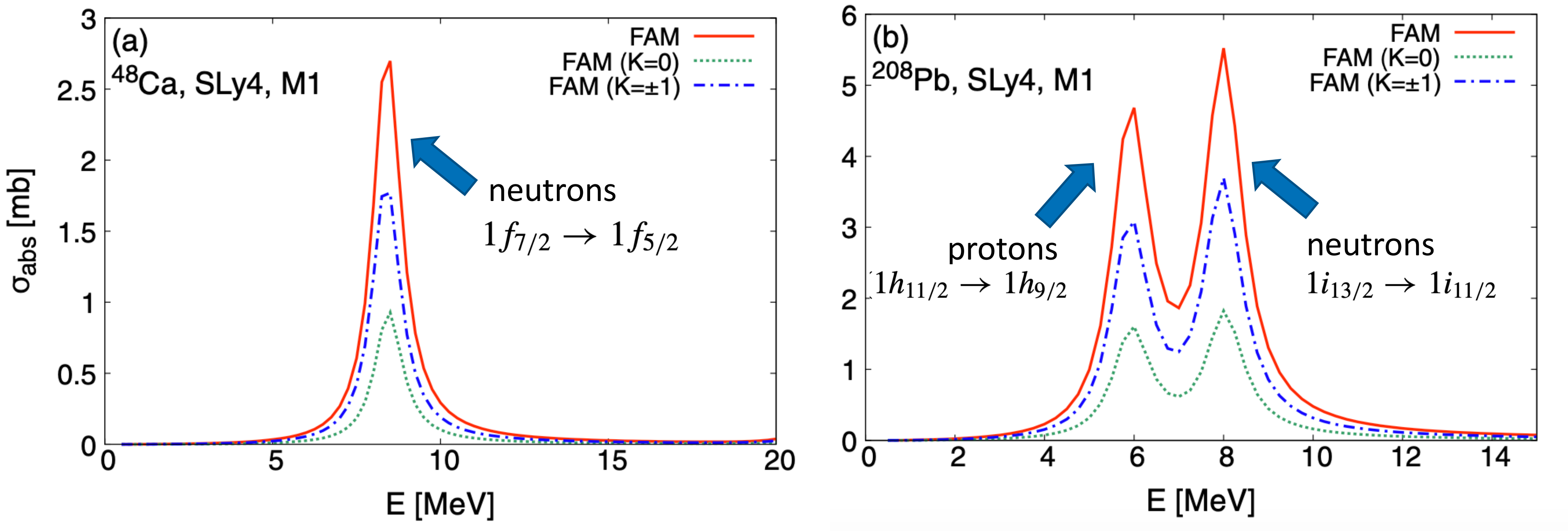
M1 transitions for double magic nuclei

The spin part mainly contributes to the M1 transitions

$$M_K = \mu_N \sum_{i=1}^{A} \left(\underline{g_s^{(i)} \frac{\vec{\sigma}_i}{2}} + g_l^{(i)} \vec{l}_i \right) \cdot \nabla (r_i Y_{1K}(\theta_i, \varphi_i)) \qquad (K = 0, \pm 1)$$

Spin part

H. Sasaki, T. Kawano and I. Stetcu, PRC105, 044311(2022)



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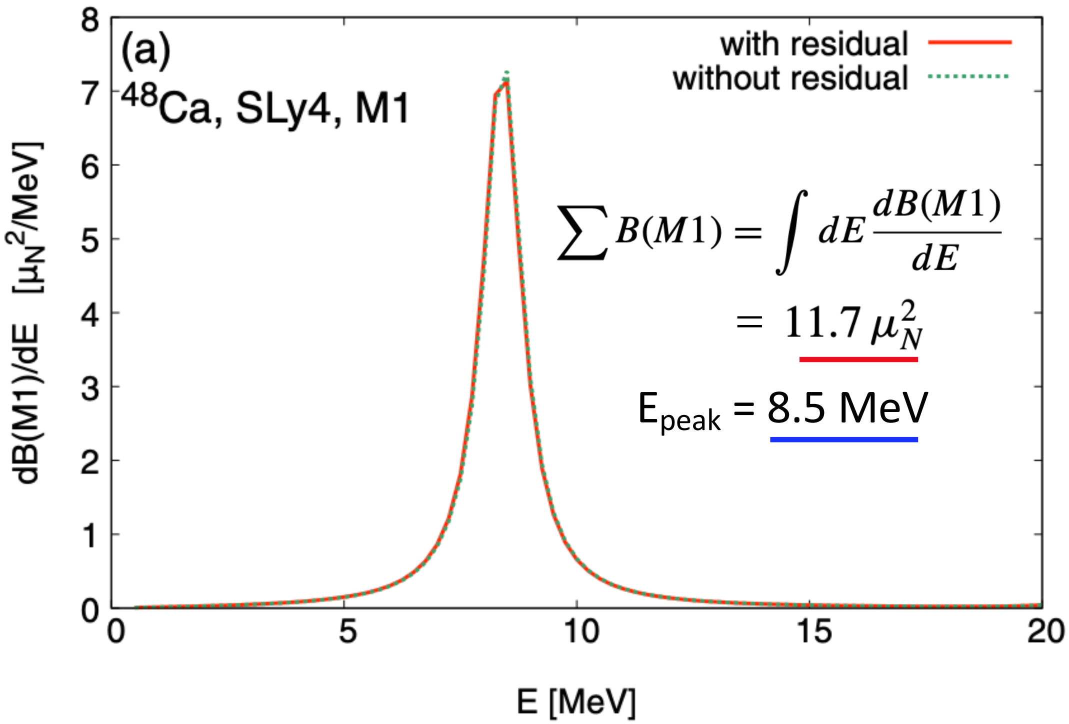
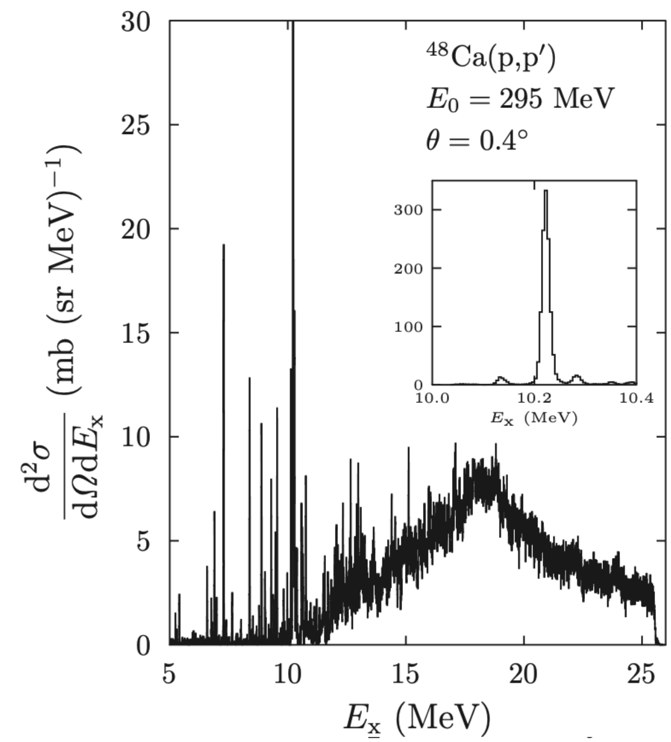
Comparison with experimental data

J. Birkhan et al., PRC93, 041202(R)(2016)



$$\sum B(M1) = \int dE \frac{dB(M1)}{dE} = 11.7\ \mu_N^2$$

E_{peak} = 8.5 MeV



$$\sum B(M1) = 3.85\text{–}4.63\ \mu_N^2$$

E_{peak} = 10.23 MeV

$$M_K = \mu_N \sum_{i=1}^{A} \left(g_s^{(i)} \frac{\vec{\sigma}_i}{2} + g_l^{(i)} \vec{l}_i \right) \cdot \nabla (r_i Y_{1K}(\theta_i, \varphi_i))$$

Some spin terms neglected in our residual interaction may upshift E_{peak}

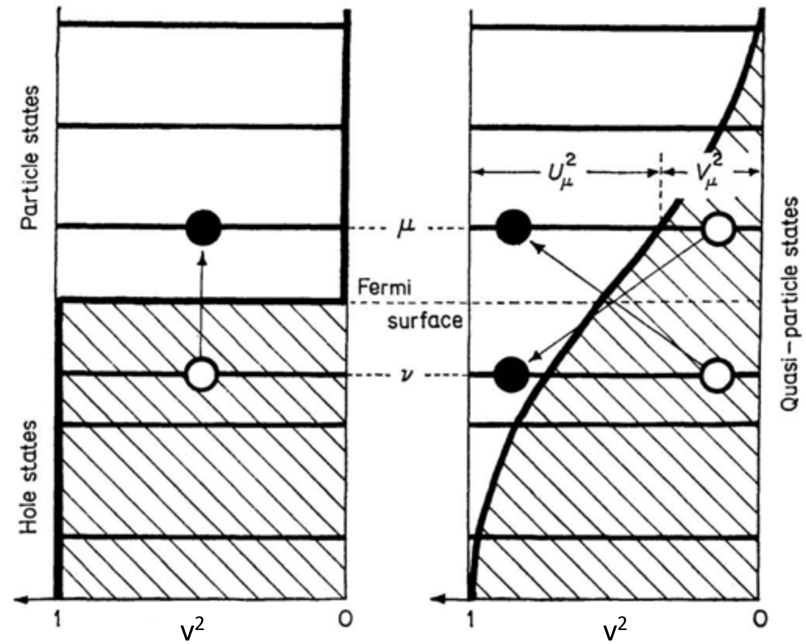
A quenching of $g_s^{(i)}$ ($g_s^{(i)} \rightarrow \sim 0.6\, g_s^{(i)}$) reduces $\sum B(M1)$

Update from RPA to Quasiparticle-RPA (QRPA)

In general, RPA calculations are failed in open-shell and deformed nuclei

The pairing force makes Fermi surface ambiguous. Then, we can not distinguish hole and particle states clearly as done in RPA.

In QRPA calculation, we update matrix elements with BCS parameters, U and V



RPA → QRPA

$$f \rightarrow U^\dagger f V^* - V^\dagger f^T U^*$$

$$\delta h \rightarrow U^\dagger \delta h V^* - V^\dagger \delta\Delta^{(-)*} V^* + U^\dagger \delta\Delta^{(+)} U^* - V^\dagger \delta h^T U^*$$

P. Avogadro and T. Nakatsukasa, PRC84, 014314(2011)

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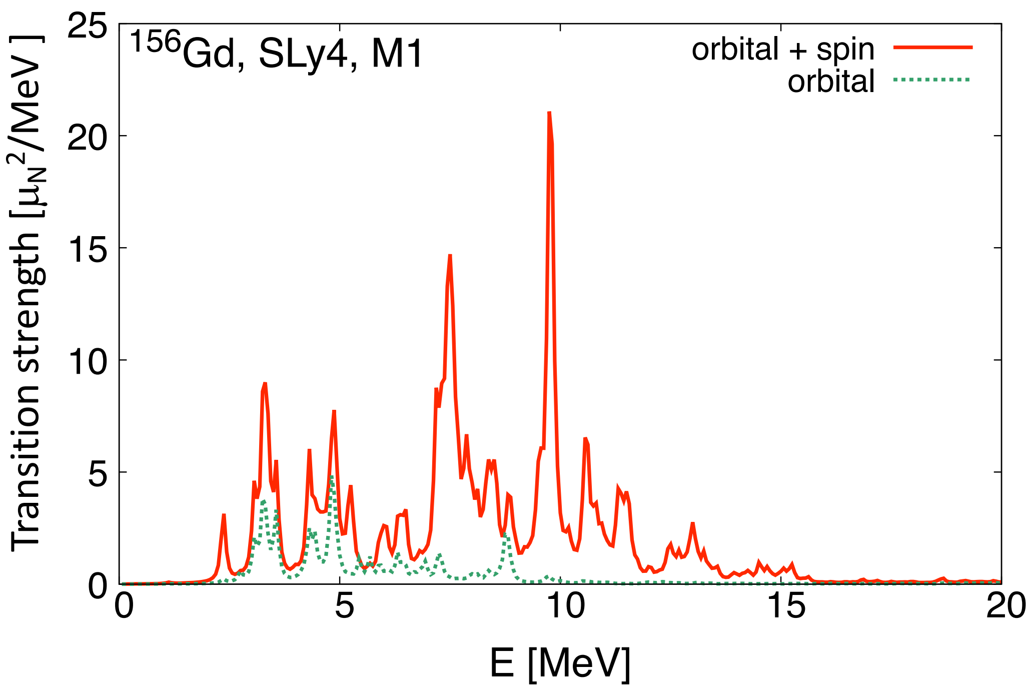
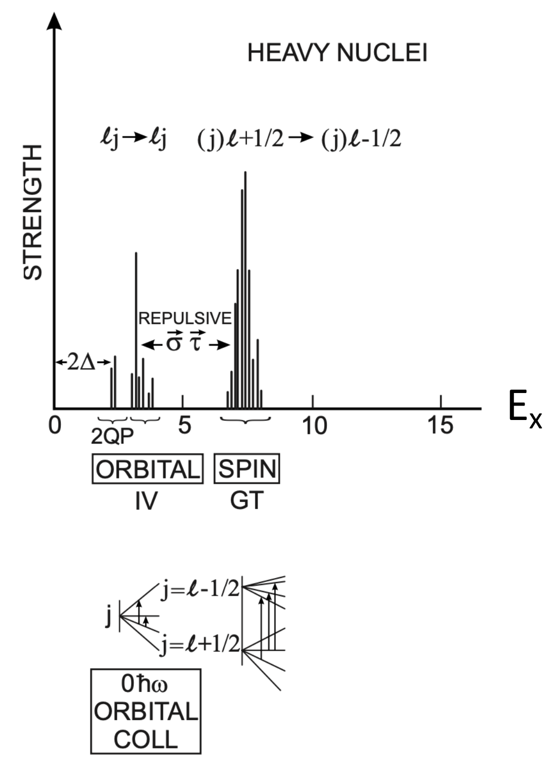
The M1 transition

The giant resonances are characterized by various oscillation modes (E1,M1,E2,M2,..)

H. Sasaki, T. Kawano and I. Stetcu, arXiv:2211.15935





K. Heyde et al., Rev. Mod. Phys.82, 2010

M1 operator

$$M_K = \mu_N \sum_{i=1}^{A} \left(\underbrace{g_s^{(i)} \frac{\vec{\sigma}_i}{2}}_{\text{spin}} + \underbrace{g_l^{(i)} \vec{l}_i}_{\text{orbital}} \right) \cdot \nabla (r_i Y_{1K}(\theta_i, \varphi_i))$$

spin orbital

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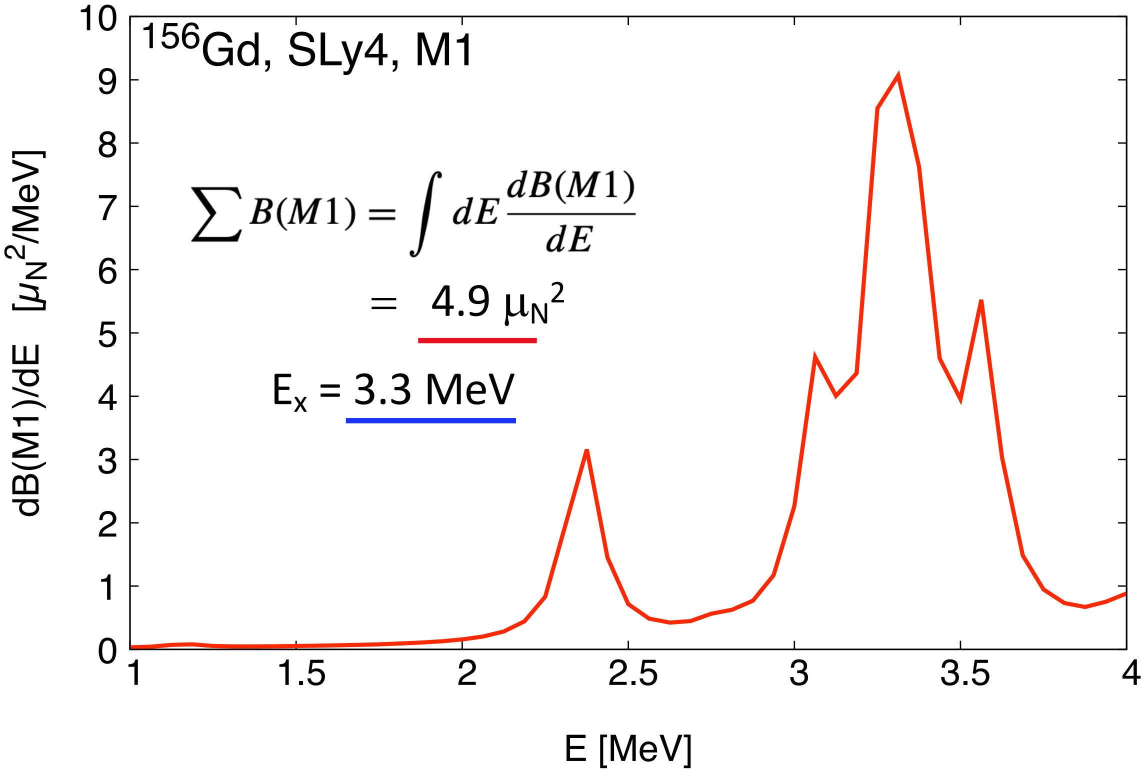
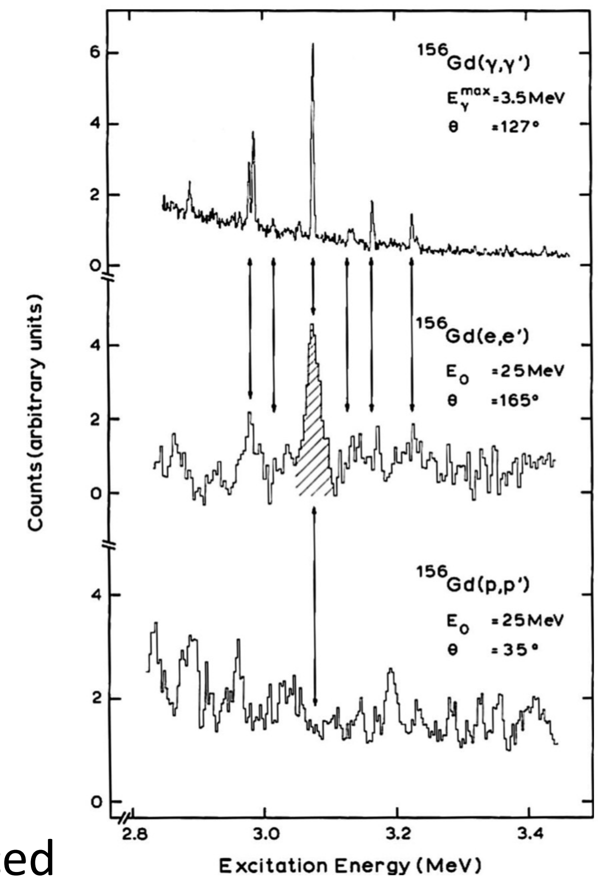
Comparison with experimental data

A. Richter, Nucl.Phys.A507,99c (1990)



$$\sum B(M1) = \int dE \frac{dB(M1)}{dE} = 4.9\ \mu_N^2$$

E_x = 3.3 MeV



$\sum B(M1) \sim 3\ \mu_N^2$

E_x = 3.1 MeV

The excitation energy of the scissors mode, E_x is reproduced but the transition strength becomes larger

→ We need a quenching of $g_s^{(i)}$?

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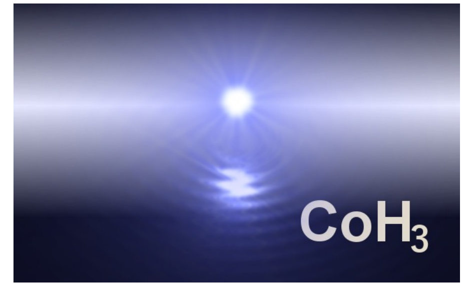
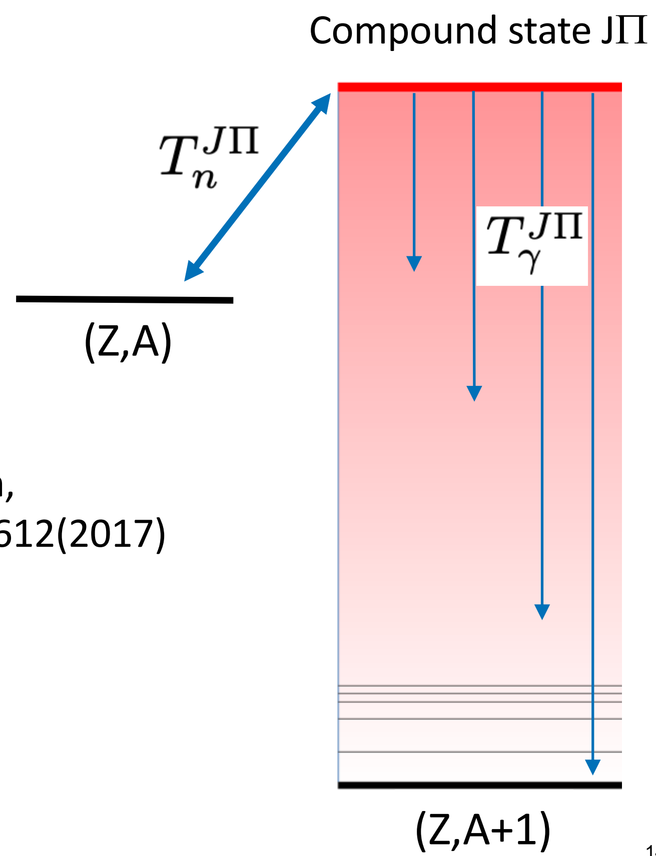
The application to neutron capture reactions

We calculate neutron capture reactions with Coupled-Channels and Hauser-Feshbach Code CoH_3



Capture cross section

$$\sigma_{n\gamma}(E_n) = \frac{\pi}{k_n^2} \sum_{J\Pi} g_c \frac{T_n^{J\Pi} T_\gamma^{J\Pi}}{T_n^{J\Pi} + T_\gamma^{J\Pi}} W_{n\gamma}^{J\Pi}$$



M.R. Mumpower, T. Kawano, J.L. Ullmann, M. Krtička, and T.M. Sprouse, PRC96,024612(2017)

Our giant resonance results are applied to calculate the γ-ray transmission coefficient $T_\gamma^{J\Pi}$

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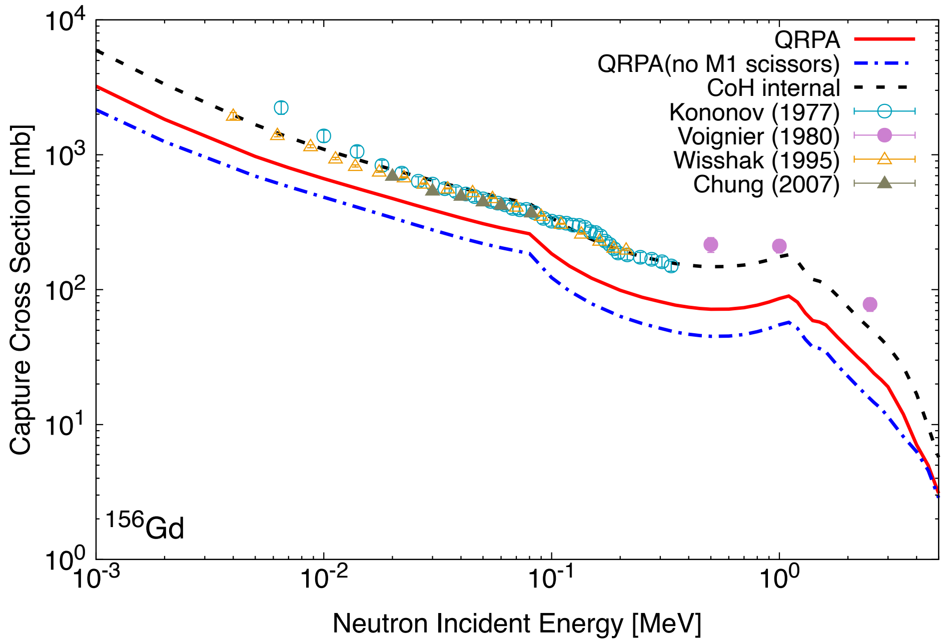
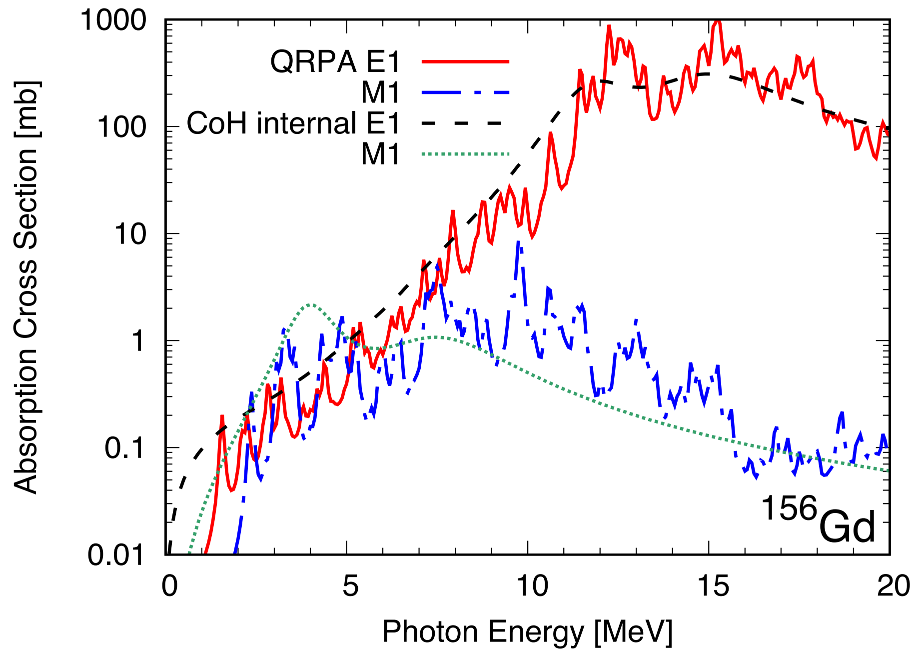
Calculated neutron capture cross sections

H. Sasaki, T. Kawano and I. Stetcu, arXiv:2211.15935





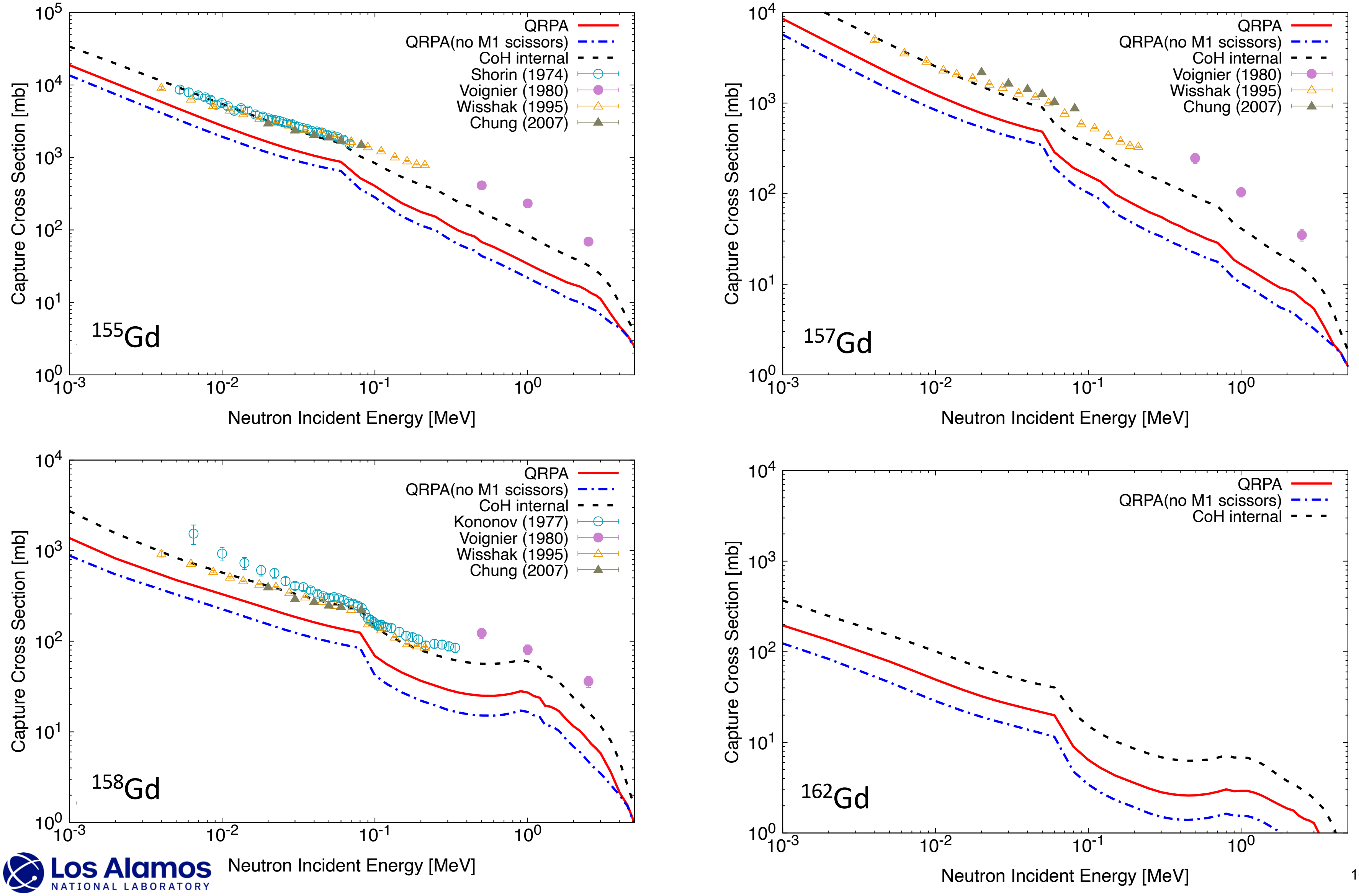
The M1 scissors mode contributes to about 40% of the total cross section

Our capture cross section (QRPA) is about half of experimental data

This discrepancy is related to small QRPA E1 in small photon energy

The capture cross sections on other Gd isotopes

H. Sasaki, T. Kawano and I. Stetcu, arXiv:2211.15935



Summary

- We rederive (Q)RPA matrices with noniterative FAM
- Our microscopic calculation of RPA well reproduces the resonance energy of E1 for heavy nuclei without any adjustment
- We reproduce a scissors mode of M1 transitions for ^{156}Gd
- We calculate (n,γ) reactions with the GR results. Our capture cross section is about half of the experimental data